Decision Problems for Subclasses of Rational Relations over Finite and Infinite Words

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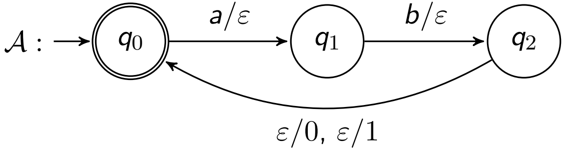
Rational Relations

- relations over finite words
- defined by transducers (non-deterministic multi-tape "automata")
- Example:



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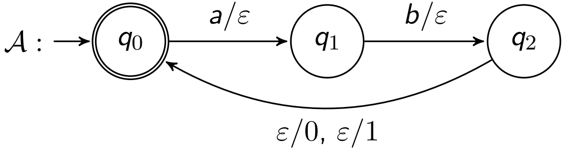

- $R_*(\mathcal{A}) = \{((ab)^n, v) \mid n \in \mathbb{N}, v \in \{0,1\}^n\}$
- e.g. $(abab, 10), (abab, 00) \in R_*(\mathcal{A})$

ω-Rational Relations

- relations over infinite words
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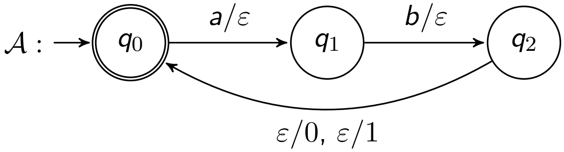

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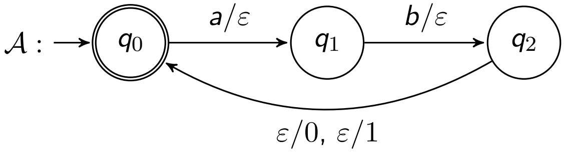

- Büchi acceptance condition (visit an accepting state infinitely often)

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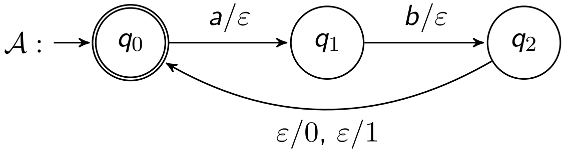

- Büchi acceptance condition (visit an accepting state infinitely often)
- $R_\omega(\mathcal{A}) = \{((ab)^\omega, v) \mid v \in \{0,1\}^\omega\}$
- e.g. $((ab)^\omega, 0^\omega), ((ab)^\omega, (10)^\omega) \in R_\omega(\mathcal{A})$

Outline

The equivalence problem for transducers

$$R_\omega(\mathcal{A}) \stackrel{?}{=} R_\omega(\mathcal{B})$$

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Deciding recognizability for (ω-)automatic relations

$$R \stackrel{?}{=} \bigcup_{i=1}^{n} L_i \times K_i$$

The Equivalence Problem

	finite words	infinite words
Given:	$R_*(\mathcal{A}) \stackrel{?}{=} R_*(\mathcal{B})$	$R_\omega(\mathcal{A}) \stackrel{?}{=} R_\omega(\mathcal{B})$
transducers $\mathcal{A}, \mathcal{B}$		

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transducers $\mathcal{A}, \mathcal{B}$	undecidable	

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transducers $\mathcal{A}, \mathcal{B}$	undecidable	undecidable
deterministic transducers $\mathcal{A}, \mathcal{B}$	decidable	

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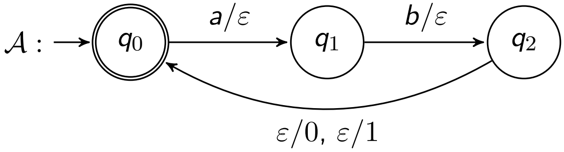
transducers $\mathcal{A}, \mathcal{B}$	undecidable	undecidable
deterministic transducers $\mathcal{A}, \mathcal{B}$	decidable	undecidable

Theorem (this work)

The equivalence problem for deterministic Büchi transducers is undecidable.

Deterministic (ω-)Rational Relations

deterministic (Büchi) transducers



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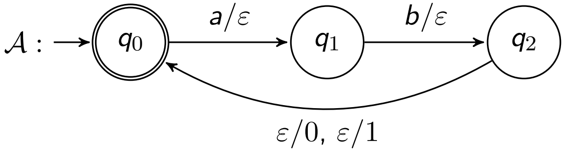

❶ state determines the component to be read

❷ state and letter determine the next state

The Equivalence Problem

$\mathcal{A}, \mathcal{B}$ deterministic (Büchi) transducers

Aim

$$R_\omega(\mathcal{A}) \stackrel{?}{=} R_\omega(\mathcal{B})$$

undecidable

The Equivalence Problem

$\mathcal{A}, \mathcal{B}$ deterministic (Büchi) transducers

Aim	Known
$R_\omega(\mathcal{A}) \stackrel{?}{=} R_\omega(\mathcal{B})$	$R_*(\mathcal{A}) \stackrel{?}{=} R_*(\mathcal{B})$
undecidable	decidable

The Equivalence Problem

$\mathcal{A}, \mathcal{B}$ deterministic (Büchi) transducers

Known		Aim	Known
$R_*(\mathcal{A}) \cap R_*(\mathcal{B}) \stackrel{?}{=} \emptyset$	$\leq$	$R_\omega(\mathcal{A}) \stackrel{?}{=} R_\omega(\mathcal{B})$	$R_*(\mathcal{A}) \stackrel{?}{=} R_*(\mathcal{B})$
undecidable		undecidable	decidable

The Equivalence Problem

$\mathcal{A}, \mathcal{B}$ deterministic (Büchi) transducers

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$R_*(\mathcal{A}) \cap R_*(\mathcal{B}) \stackrel{?}{=} \emptyset \quad \leq$	$R_\omega(\mathcal{A}) \stackrel{?}{=} R_\omega(\mathcal{B})$	$R_*(\mathcal{A}) \stackrel{?}{=} R_*(\mathcal{B})$
undecidable	undecidable	decidable

$$(\mathcal{A}, \mathcal{B}) \mapsto (\mathcal{A}', \mathcal{B}')$$

$$\rightsquigarrow \; R_*(\mathcal{A}) \cap R_*(\mathcal{B}) \neq \emptyset \Leftrightarrow R_\omega(\mathcal{A}') \neq R_\omega(\mathcal{B}')$$

The reduction is based on an idea of Böhm, Göller, Halfon, and Hofman 2017

The Equivalence Problem — Reduction

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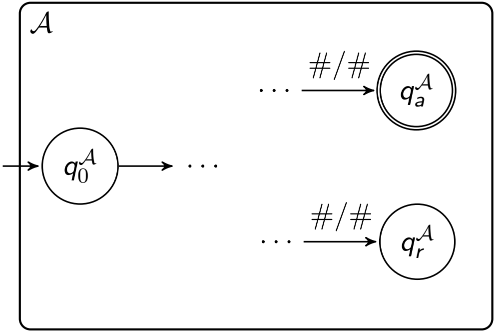

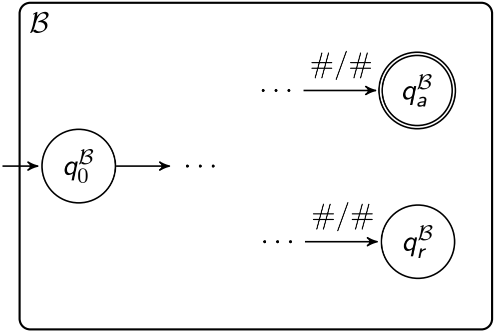

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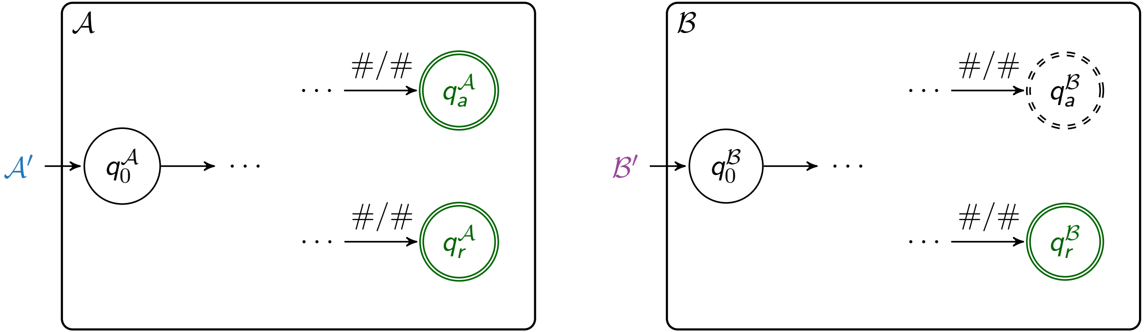

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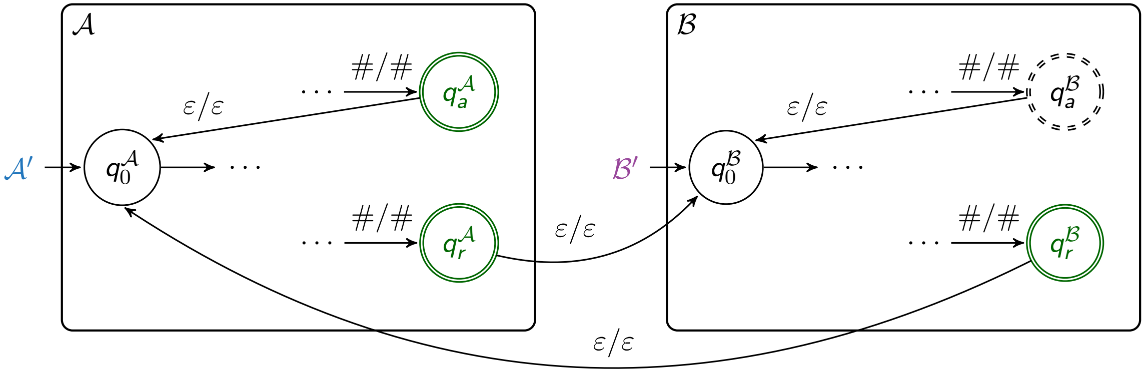

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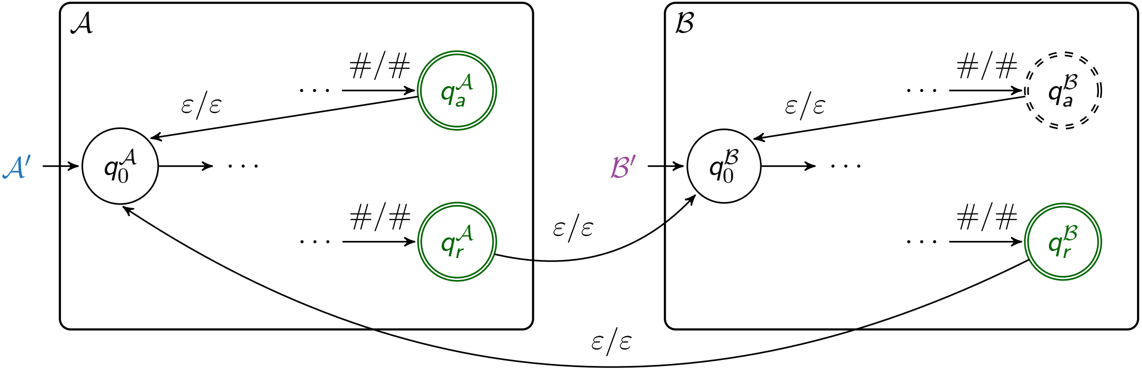
$$(u\#, v\#) \in R_*(\mathcal{A}) \cap R_*(\mathcal{B}) \;\Rightarrow\; (u\#, v\#)^\omega \in R_\omega(\mathcal{A}') \setminus R_\omega(\mathcal{B}')$$



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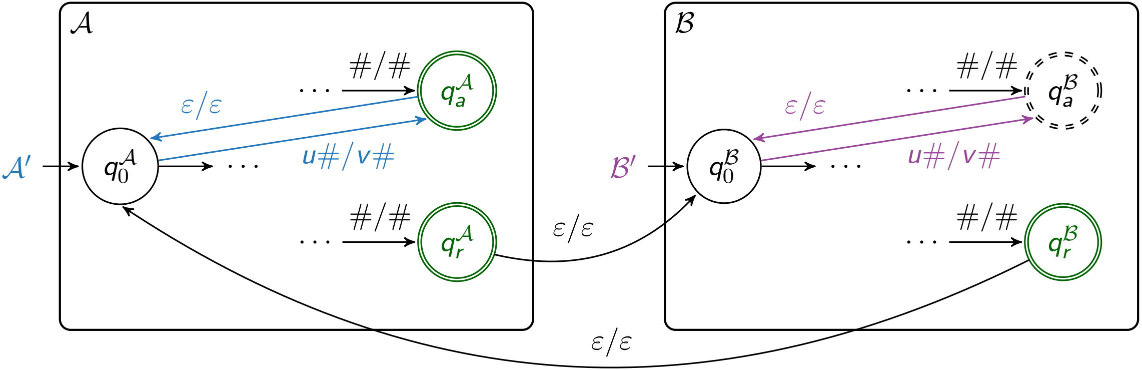

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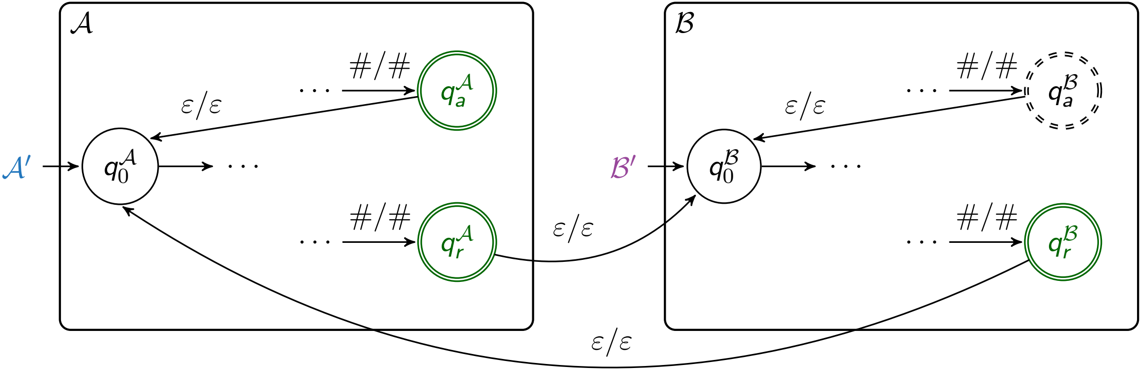
$$(u_1\#, v_1\#)(u_2\#, v_2\#)\ldots \in R_\omega(\mathcal{A}') \setminus R_\omega(\mathcal{B}') \;\Rightarrow\; \exists k : (u_k\#, v_k\#) \in R_*(\mathcal{A}) \cap R_*(\mathcal{B})$$



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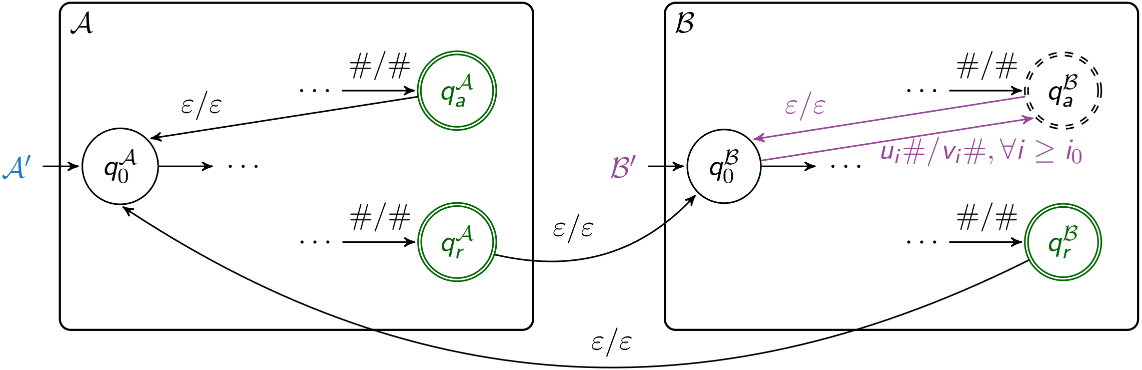

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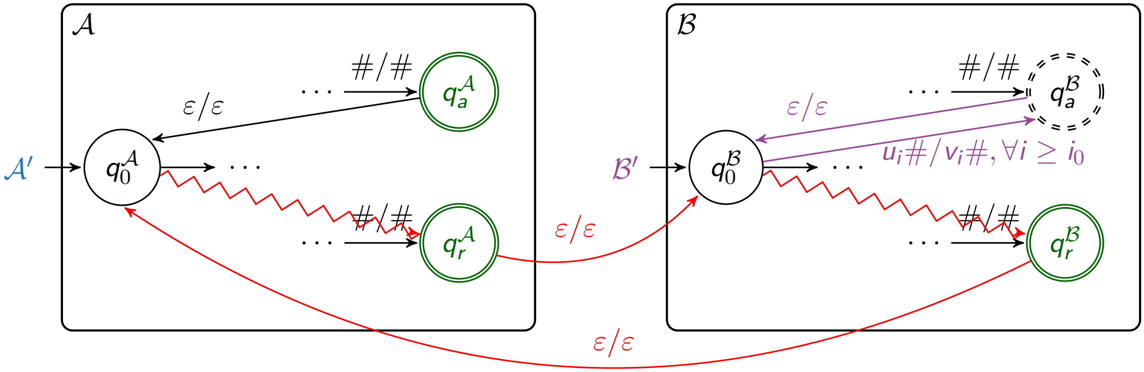

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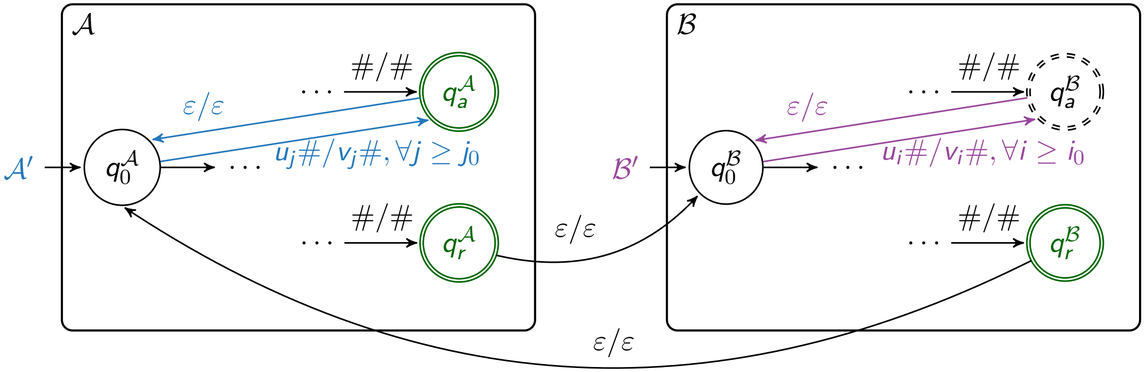

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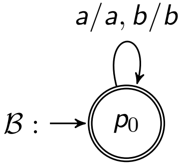
Deciding recognizability for (ω-)automatic relations

$$R \stackrel{?}{=} \bigcup_{i=1}^{n} L_i \times K_i$$

(ω-)Automatic Relations

- synchronous transducer (single reading head)
- “automaton over a product alphabet”



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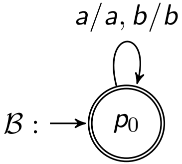

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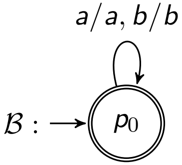

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- finite words may be of different length

(ω-)Recognizable Relations

$$R = \bigcup_{i=1}^{n} L_i \times K_i$$

(L_i and K_i are) (ω-)regular languages

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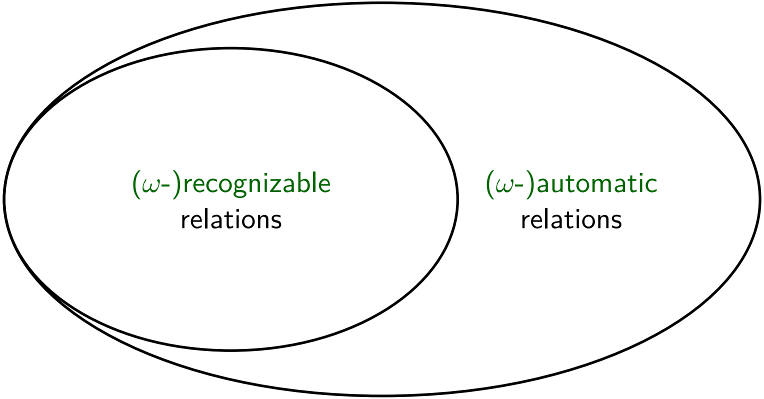
(ω-)regular languages

Example: $R = a^\omega \times a^\omega \;\cup\; b^\omega \times (a+b)^\omega$

(ω-)Rational Relations – Hierarchy



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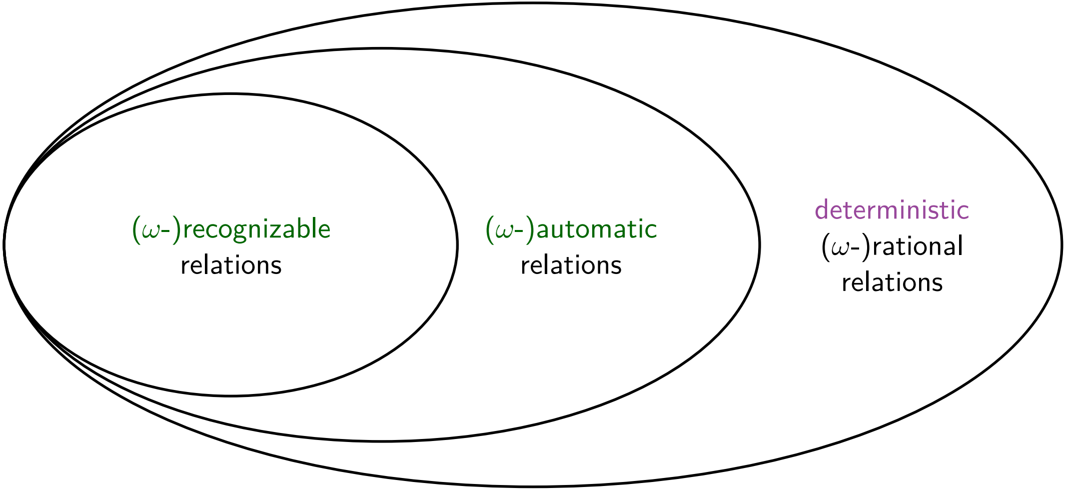

Deciding Recognizability of (ω-)Automatic Relations

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Theorem (Carton, Choffrut, and Grigorieff 2006)

Recognizability for automatic relations is decidable in doubly exponential time.

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Theorem (this work)

Recognizability is decidable for

1. *ω-automatic relations in doubly exponential time,*
2. *binary automatic relations in exponential time.*

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Deciding recognizability

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R ω-automatic	R binary automatic
2EXPTIME	EXPTIME

Literature

- Böhm, Stanislav, Stefan Göller, Simon Halfon, and Piotr Hofman (2017). "On Büchi One-Counter Automata". In: 34th Symposium on Theoretical Aspects of Computer Science (STACS 2017). Ed. by Heribert Vollmer and Brigitte Vallée. Vol. 66. Leibniz International Proceedings in Informatics (LIPIcs). Dagstuhl, Germany: Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, 14:1–14:13.
- Carton, Olivier, Christian Choffrut, and Serge Grigorieff (2006). "Decision problems among the main subfamilies of rational relations". In: RAIRO-Theoretical Informatics and Applications 40.02, pp. 255–275.