Parallel-Correctness and Parallel-Boundedness for Datalog Programs

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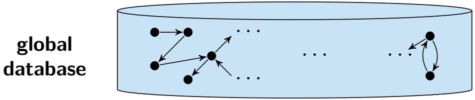
22nd ICDT March 28, 2019 Lisbon

Distributed Evaluation



Query

- transitive closure T
- Datalog program

$$T(x,y) \leftarrow E(x,y).$$
$$T(x,z) \leftarrow T(x,y),\ E(y,z).$$

- recursive evaluation
  (fixed point computation)

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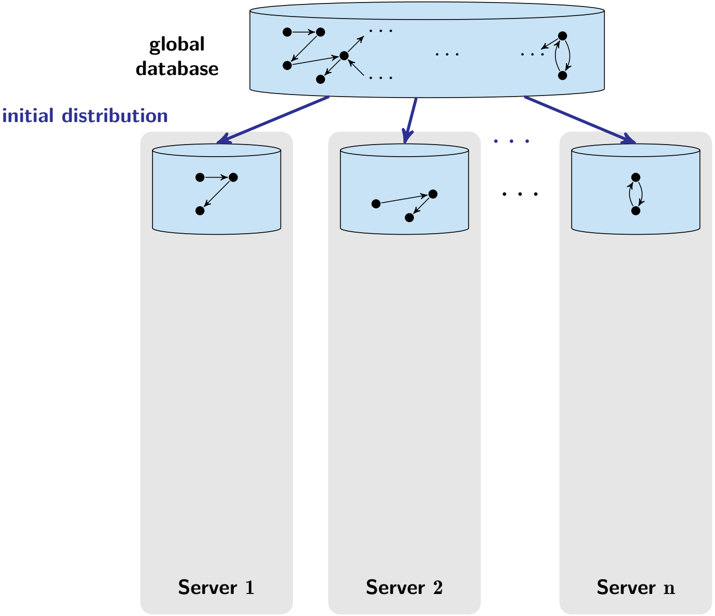

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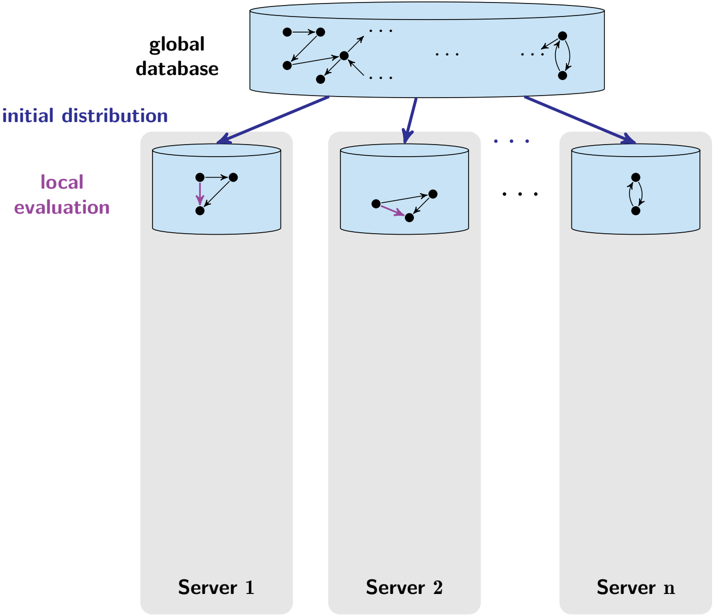

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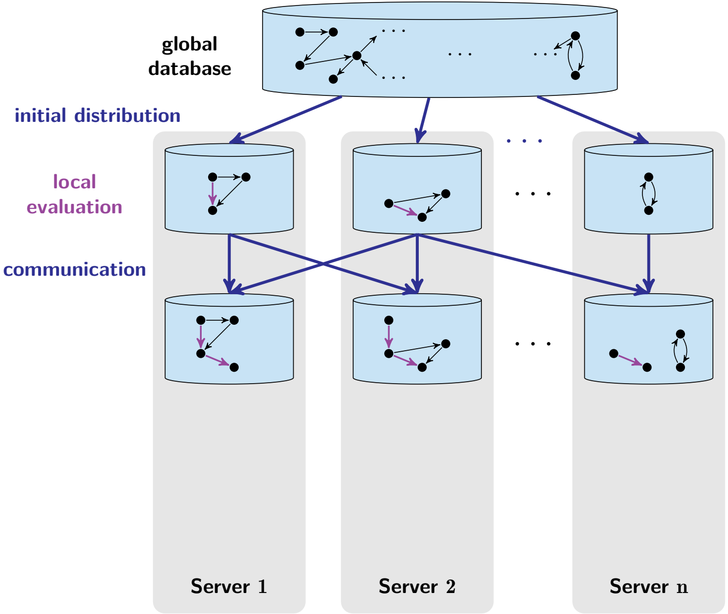

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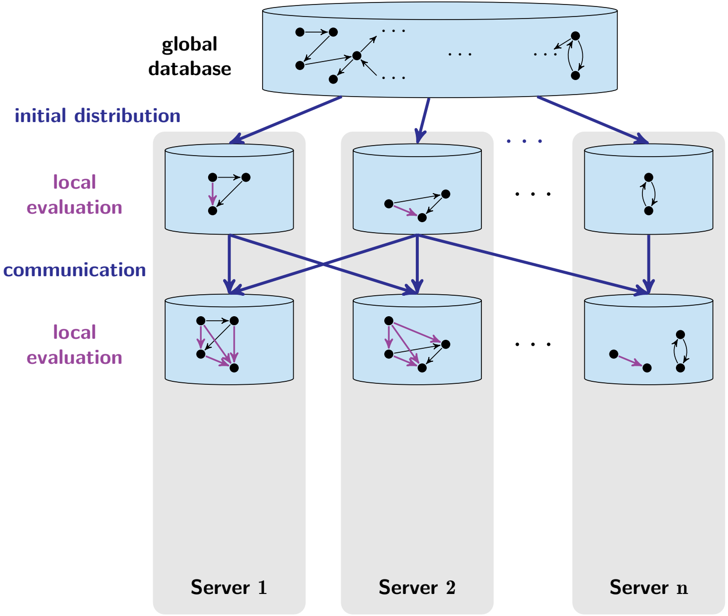

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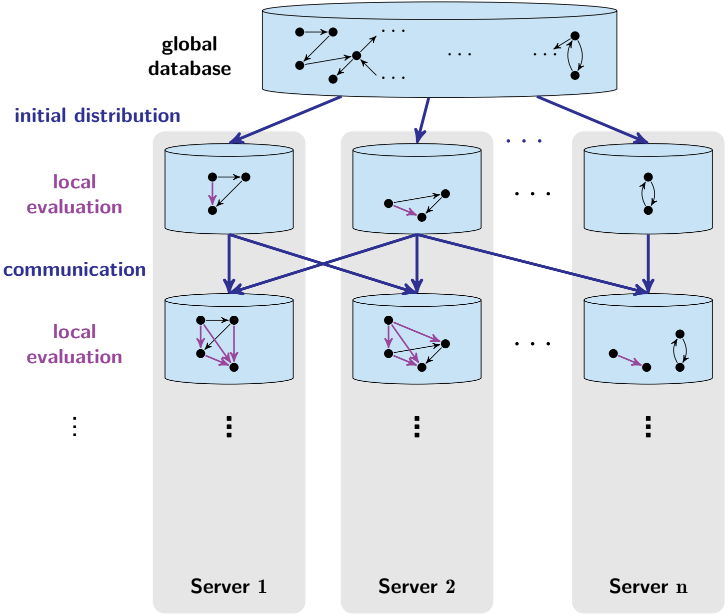

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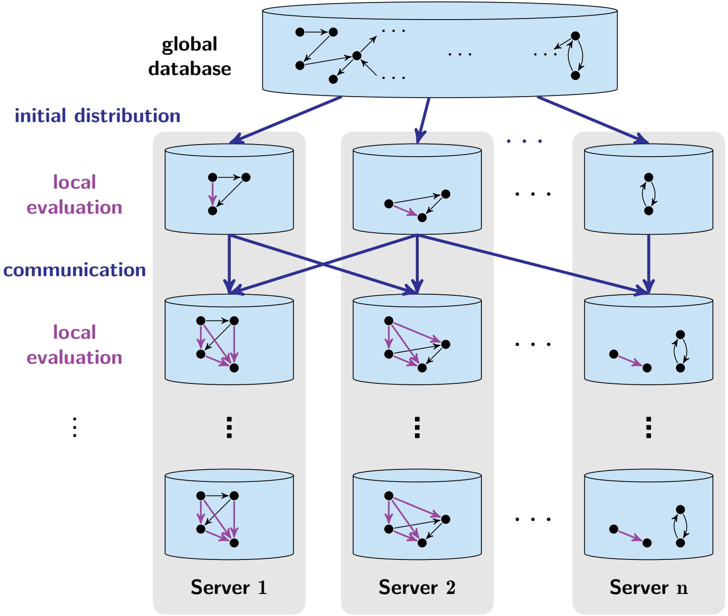

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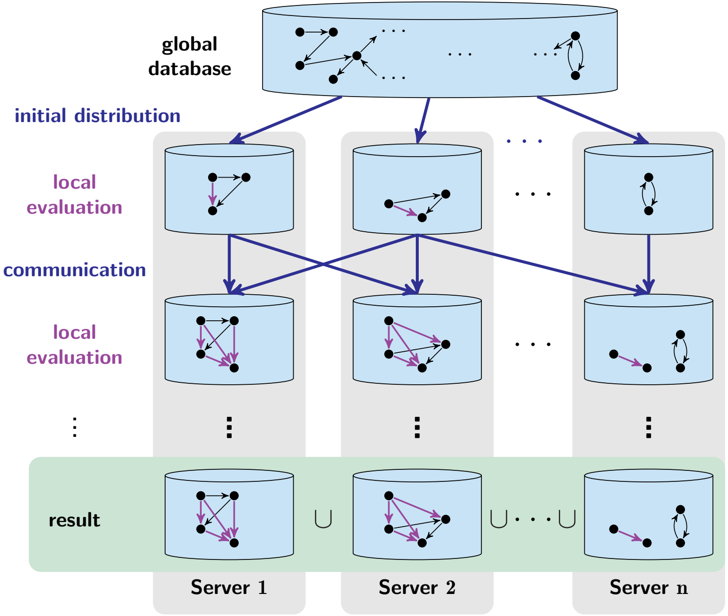
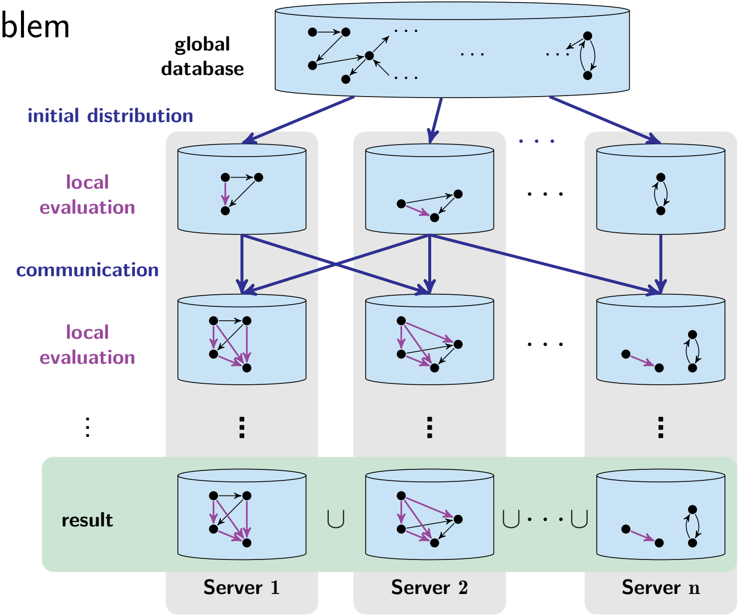


The Parallel-Correctness Problem



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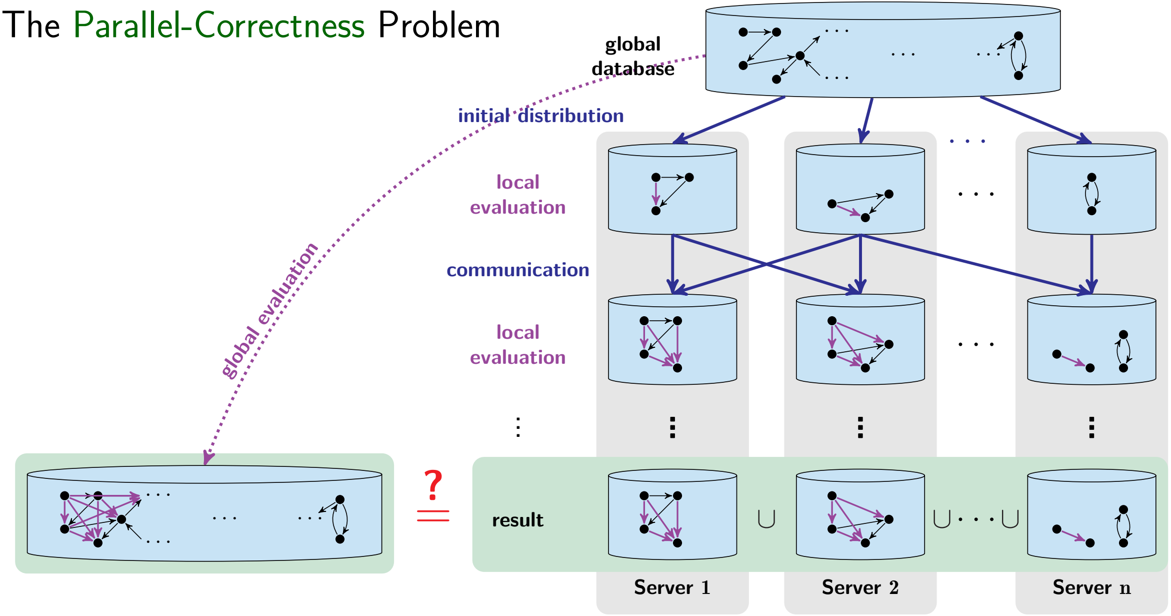

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- communication policy

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Do distributed and global evaluation yield the same result for all databases?

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Parallel-correctness for general Datalog programs is undecidable.

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Parallel-correctness for general Datalog programs is undecidable.

- Even for “simple” policies:
  - only two servers
  - all but one relations are distributed to both servers
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- Is there a fragment of Datalog for which parallel-correctness is decidable?

Basics

Relational databases

$\underbrace{E(a,b,c)}_{\text{fact}}, E(a,d,g), F(a,d), \dots$

with extensional relation symbols $E, F, \dots$

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with extensional relation symbols $E, F, \ldots$

Datalog programs consist of rules

$$\underbrace{T(x,y)}_{\text{head}} \leftarrow \underbrace{E(x,y,z), \overbrace{R(x,v)}^{\text{atom}}}_{\text{body}}.$$

head atoms are intensional (i.e. not extensional)

Parallel-Correctness and Containment

Undecidability of parallel-correctness results from the containment problem

… and containment is undecidable for general Datalog

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general Datalog

$\subsetneq$

monadic Datalog	only unary head atoms	Example: $R(x) \leftarrow S(x), E(y, z, u).$

Containment is decidable for monadic Datalog

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general Datalog

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frontier-guarded Datalog

Each rule has a **guard** atom

- contains all head variables
- extensional

Example:

$T(x, y) \leftarrow \mathbf{E}(\mathbf{x}, \mathbf{y}, \mathbf{z}), F(y, v).$

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frontier-guarded Datalog

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Example:
$T(x, y) \leftarrow \mathbf{E}(\mathbf{x}, \mathbf{y}, \mathbf{z}), F(y, v).$

rewrite

monadic Datalog

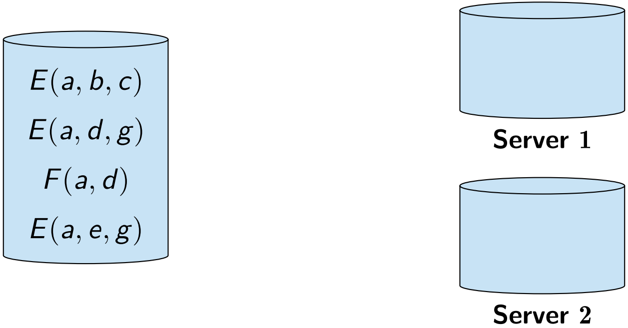
only unary head atoms

Example:
$R(x) \leftarrow S(x), E(y, z, u).$

Containment is decidable for monadic Datalog and frontier-guarded Datalog

Distribution Policies

Idea: Use hash functions $h_1, \ldots, h_k$ fast, evenly distribution



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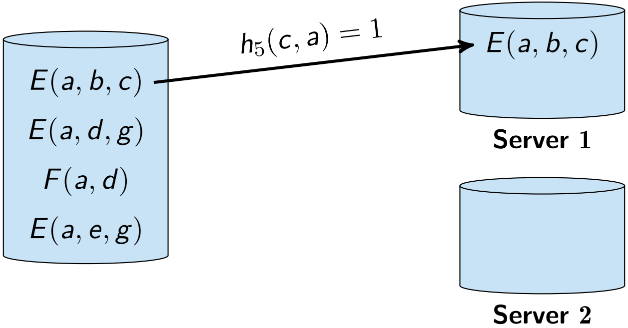

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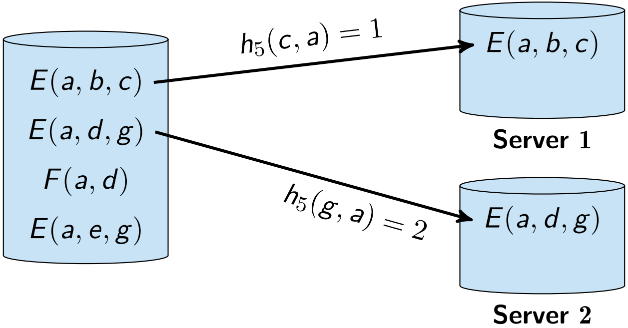

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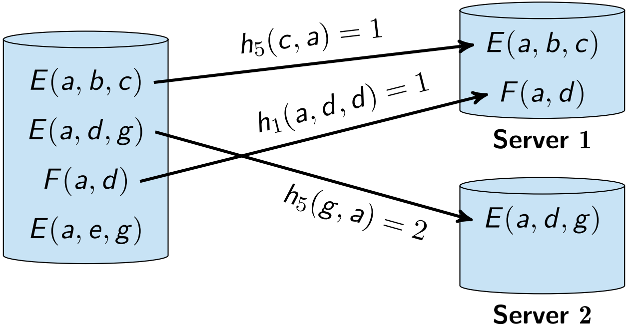

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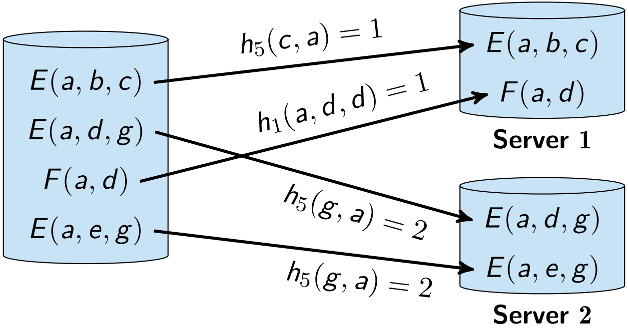

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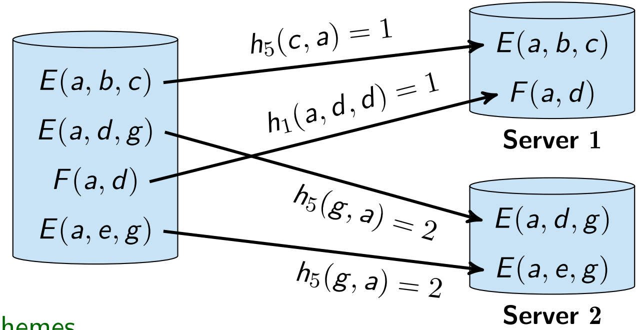

Here: Hash policy schemes

- describes how hash functions are applied
- defines class of hash functions

Communication Policies

Data-Moving Distribution Constraints

$$\underbrace{R(x, y)@\lambda, S(y)@\kappa}_{\text{body}} \rightarrow \underbrace{R(x, y)@\kappa}_{\text{head}}$$

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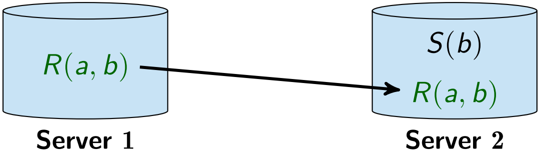


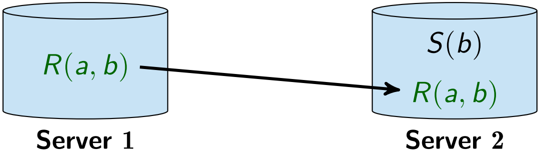
Communication Policies

Data-Moving Distribution Constraints

$$\underbrace{R(x,y)@\lambda, S(y)@\kappa}_{\text{body}} \rightarrow \underbrace{R(x,y)@\kappa}_{\text{head}}$$

Both $R(x,y)$ and κ occur in the body.

- No creation of facts
- No creation of servers



Parallel-Correctness

Theorem

Parallel-correctness for monadic and frontier-guarded Datalog,

- *hash policy schemes, and*
- *data-moving distribution constraints*

is undecidable.

A Decidable Variant

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Parallel-correctness for frontier-guarded Datalog,

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is 2ExpTime-complete.

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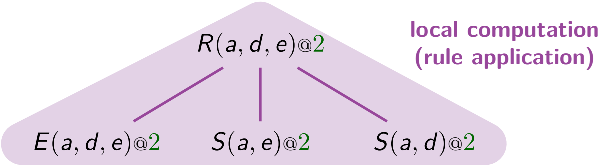
Lower bound: reduction from the containment problem

Distributed Proof Trees

$R(a, d, e)@2$

Distributed Proof Trees



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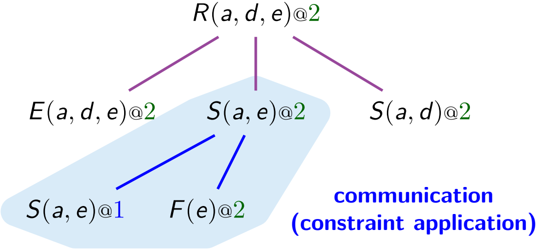

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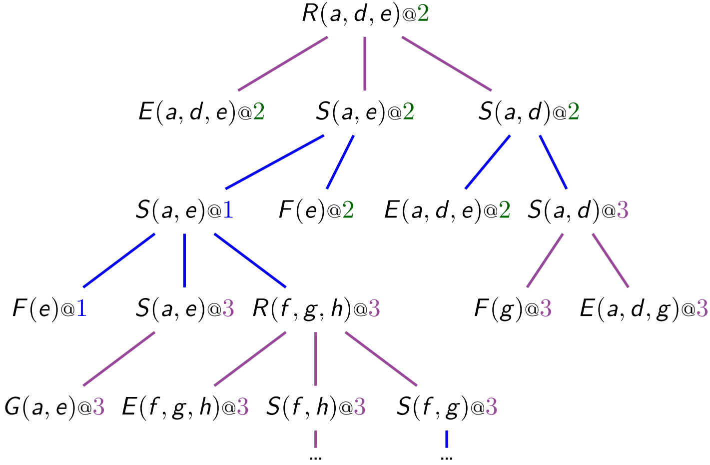

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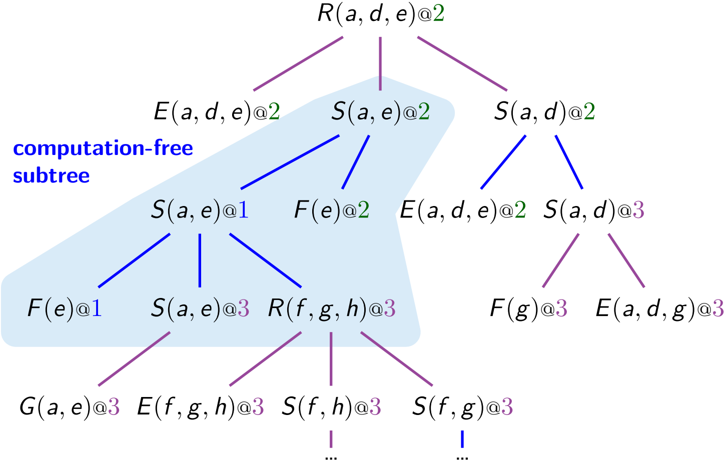

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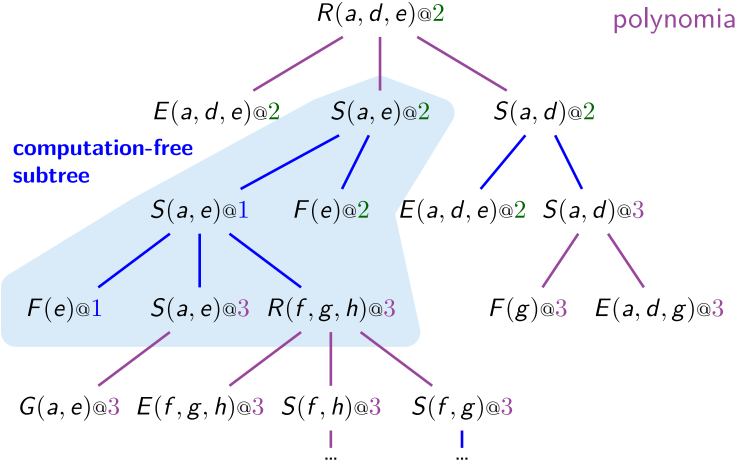

Polynomial Communication Property

The size of computation-free subtrees is polynomially bounded.

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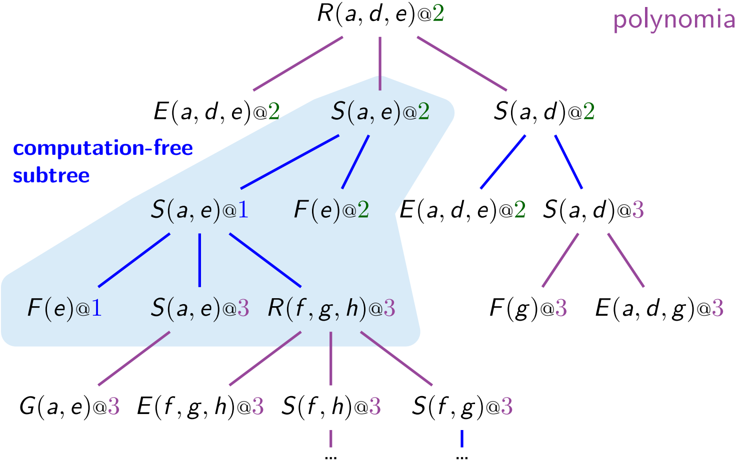

Polynomial Communication Property

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Two Settings

1. Syntactical restriction of distribution constraints
2. Changed semantics
   non-transitive setting

Proof Approach: Parallel-Correctness is in 2ExpTime

❶ Consider only worst-case hash functions

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3. Test containment

Proof Ingredient: Scattering Hash Functions

Two facts meet on the same server, if and only if,

- they were hashed by the same hash function, and
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No "accidental" collisions

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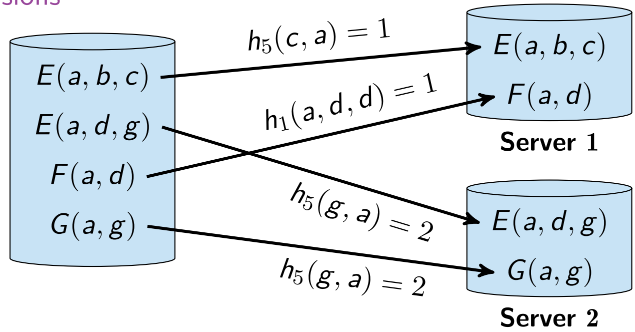

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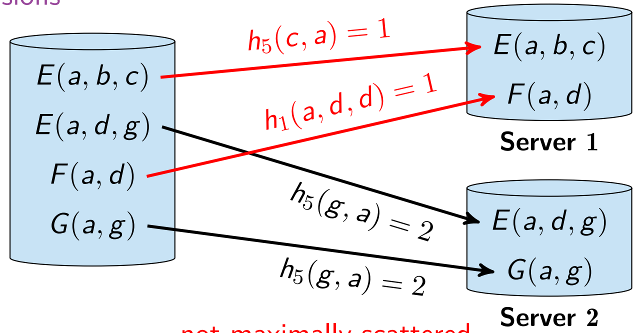

not maximally scattered

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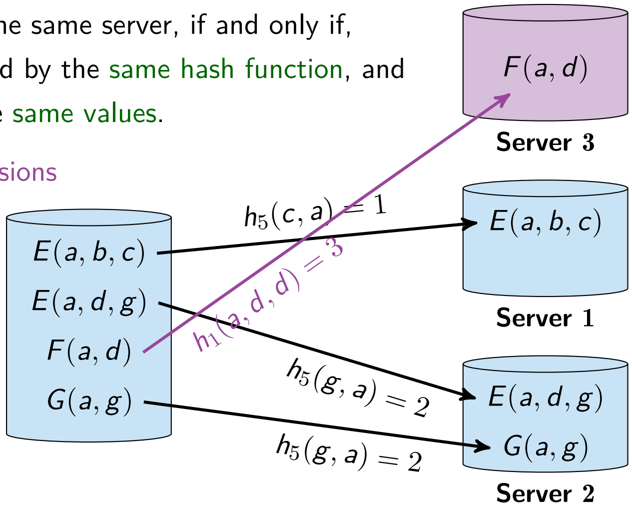

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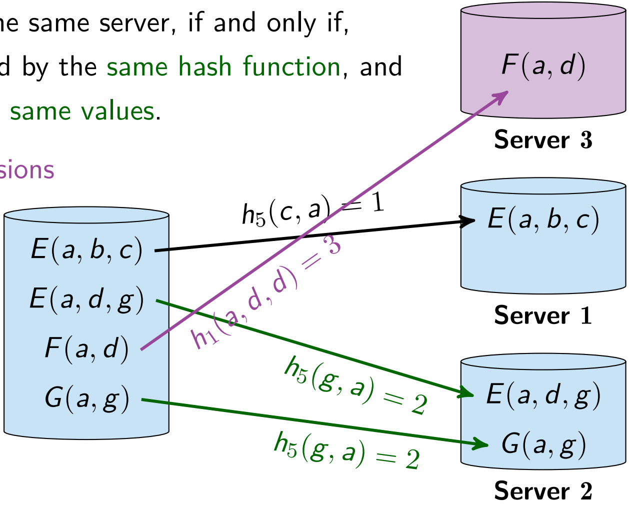

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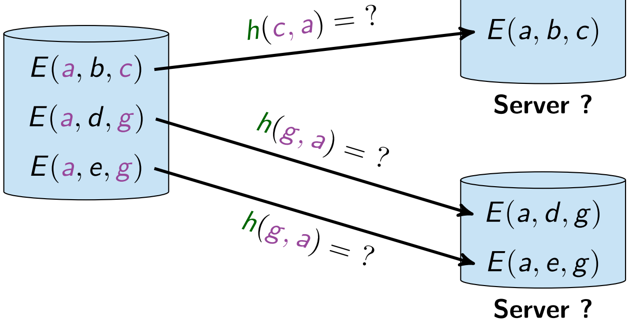

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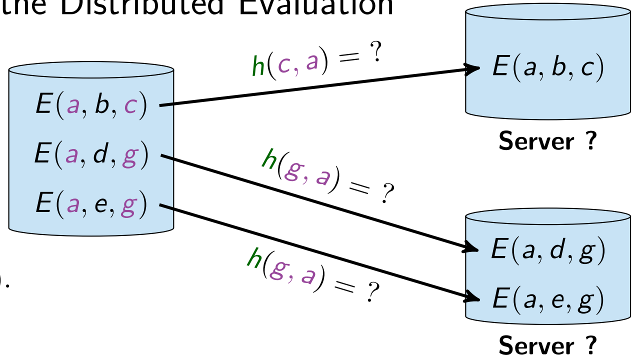

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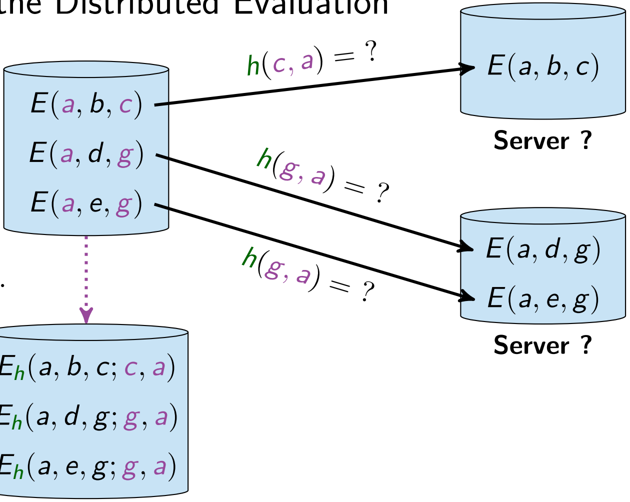

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Test containment of input program and compiled program

- The compiled program has exponential size
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  … but each rule has polynomial size

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  … but only doubly exponential in the size of rules

  requires very technical analysis

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In the non-transitive setting, parallel-correctness for frontier-guarded Datalog,

- *hash policy schemes, and*
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is 2ExpTime-*complete.*

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Corollary

In the non-transitive setting, parallel-correctness for monadic Datalog,

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C[illegible]llary

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Parallel-correctness for frontier-guarded Datalog,

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Two settings with polynomial communication property:

❶ modest data-moving distribution constraints

❷ non-transitive setting

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- Using hash policy schemes for communication also yields 2ExpTime-completeness
- Parallel-boundedness (please, ask question)

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Open Questions

- Boundary of decidability for parallel-correctness?
- Other formalism to specify policies?

Parallel-Boundedness

Parallel-boundedness

There is a bound $r \in \mathbb{N}$ such that for every database in the distributed evaluation after r communication rounds no new facts are computed.

- Local computations may be unbounded!
- Does not imply FO-definability

Classical boundedness

There is a bound $k \in \mathbb{N}$ such that for every database the fixed point computation terminates after k iterations.

- Classical boundedness implies FO-definability

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that are parallel-correct is 2ExpTime-*complete.*

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Rules for local computation

$$T(y) \leftarrow F(x, y), R(x).$$

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Rules for communication

$$T_h(y; u, v) \leftarrow F_h(x,y; u, v), \mathbf{R_h}(\mathbf{x}; \mathbf{u}, \mathbf{v}).$$

$$S(z)@\kappa, R(x)@\lambda \rightarrow \mathbf{R}(\mathbf{x})@\kappa$$

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Rules for communication

$$T_h(y; u, v) \leftarrow F_h(x, y; u, v), \mathbf{R_h}(\mathbf{x}; \mathbf{u}, \mathbf{v}). \quad S(z)@\kappa, R(x)@\lambda \rightarrow \mathbf{R}(\mathbf{x})@\kappa \xrightarrow{\text{inline}} T_h(y; u, v) \leftarrow F_h(x, y; u, v), S_h(z; u, v), R_{h'}(x; w, t).$$

Simulation of the Distributed Evaluation

Rules for local computation

$$T(y) \leftarrow F(x,y), R(x). \quad \xrightarrow[\text{and variables}]{\text{add hash index}} \quad T_h(y;u,v) \leftarrow F_h(x,y;u,v), R_h(x;u,v).$$

Rules for communication

$$\begin{array}{c} T_h(y;u,v) \leftarrow F_h(x,y;u,v), \mathbf{R_h}(\mathbf{x};\mathbf{u},\mathbf{v}). \\ S(z)@\kappa, R(x)@\lambda \rightarrow \mathbf{R}(\mathbf{x})@\kappa \end{array} \quad \xrightarrow{\text{inline}} \quad \begin{array}{c} T_h(y;u,v) \leftarrow F_h(x,y;u,v), \\ S_h(z;u,v), R_{h'}(x;w,t). \end{array}$$

For all combinations of

- hash functions and variables,
- distribution constraints **recursively** (bounded by polynomial communication property)

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Monadic Datalog: Head atoms are unary

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frontier-guarded

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Theorem

In the non-transitive setting, parallel-correctness for monadic Datalog,

- *hash policy schemes, and*
- *data-moving distribution constraints*

is undecidable.

Literature

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