Big Graph Processing Systems

Part I: Graph Query Paradigms and their Semantics

▶ **Chapter 1:** Graph Pattern Matching

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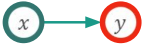
This presentation is an adaption of slides from Angela Bonifati

Graph Pattern Matching

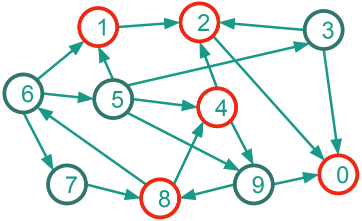
Subgraph pattern p



Graph with place holders x and y

Matching p on G

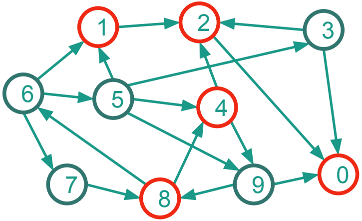
Finds all subgraphs in G that fit to p



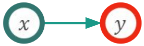
Graph Pattern Matching

Matching p on G

Finds all subgraphs in G that fit to p



Subgraph pattern p



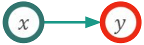
Graph with place holders x and y

Intuitively, how many matches does p have on G?

Number of Results	
A	5
B	8
C	10
D	14

Graph Pattern Matching

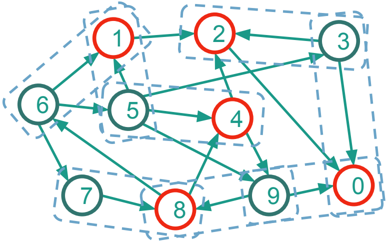
Subgraph pattern p



Graph with place holders x and y

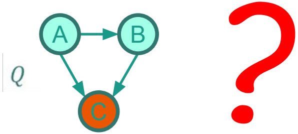
Matching p on G

Finds all subgraphs in G that fit to p



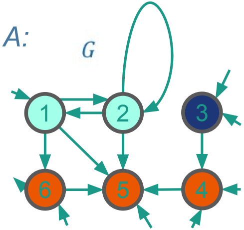
Graph Pattern Matching

Given the following query:

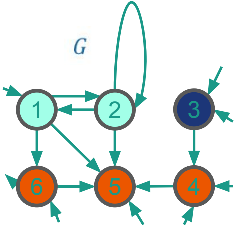


What you think is a valid match?

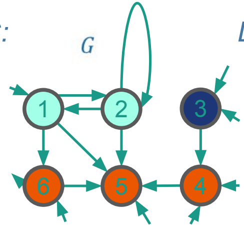
A:



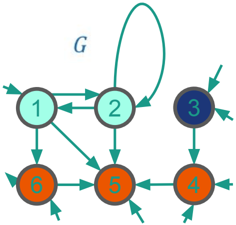
B:



C:

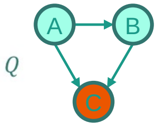


D:



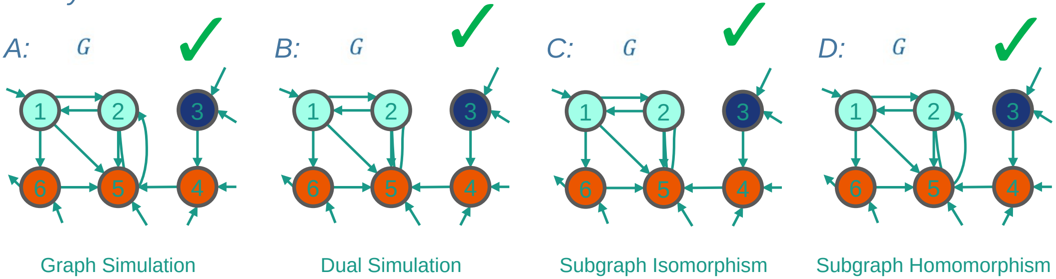
Graph Pattern Matching

Given the following query:



Well, depends on the matching semantics.

What you think is a valid match?



Matching Semantics

Plethora of Matching Semantics

[Miller et al., "Research Directions for Big Data Graph Analytics", *IEEE International Congress on Big Data, New York City, NY, USA, June 27 - July 2*, 2015]

Different Matching Semantics

Model	Complexity Class	Source	Results Contained in
Graph Simulation	Quadratic	Henzinger et al. 1995 [18]	-
Dual Simulation	Cubic	Ma et al. 2011 [19]	Graph Simulation
Strong Simulation	Cubic	Ma et al. 2011 [19]	Dual Simulation
Strict Simulation	Cubic	Fard et al., 2013 [20]	Strong Simulation
Tight Simulation	Cubic	Fard et al., 2014 [21]	Strict Simulation
CAR-Tight Simulation	Cubic	Fard et al., 2014 [22]	Tight Simulation
Graph Homeomorphism	$\mathcal{NP}$-hard	Fortune et al., 1980 [28]	-
Graph Homomorphism	$\mathcal{NP}$-hard	Hell and Nesetril, 1990 [23]	Graph Homeomorphism and Tight Simulation
Subgraph Isomorphism	$\mathcal{NP}$-hard	Garey and Johnson, 1979 [29]	Graph Homomorphism and CAR-Tight Simulation

[18] M. R. Henzinger, T. A. Henzinger, and P. W. Kopke, "Computing simulations on finite and infinite graphs," in *Foundations of Computer Science, 1995. Proceedings., 36th Annual Symposium on.* IEEE, 1995, pp. 453–462.

[19] S. Ma, Y. Cao, W. Fan, J. Huai, and T. Wo, "Capturing topology in graph pattern matching," *Proceedings of the VLDB Endowment*, vol. 5, no. 4, pp. 310–321, 2011.

[20] A. Fard, M. U. Nisar, L. Ramaswamy, J. A. Miller, and M. Saltz, "A distributed vertex-centric approach for pattern matching in massive graphs," in *Big Data Conference*, Oct 2013, pp. 403–411.

[21] A. Fard, M. U. Nisar, J. A. Miller, and L. Ramaswamy, "Distributed and scalable graph pattern matching: Models and algorithms," *International Journal of Big Data (IJBD)*, vol. 1, no. 1, 2014.

[22] A. Fard, S. Manda, L. Ramaswamy, and J. A. Miller, "Effective caching techniques for accelerating pattern matching queries," in *Big Data (Big Data), 2014 IEEE International Conference on.* IEEE, 2014, pp. 491–499.

[23] P. Hell and J. Nešetřil, "On the complexity of h-coloring," *Journal of Combinatorial Theory, Series B*, vol. 48, no. 1, pp. 92–110, 1990.

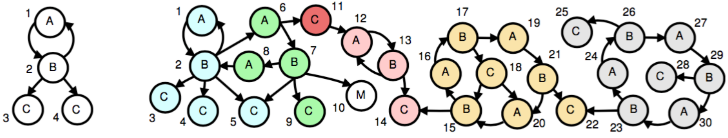
[28] S. Fortune, J. Hopcroft, and J. Wyllie, "The directed subgraph homeomorphism problem," *Theoretical Computer Science*, vol. 10, no. 2, pp. 111–121, 1980.

[29] R. G. Michael and S. J. David, "Computers and intractability: a guide to the theory of NP-completeness," *WH Freeman & Co., San Francisco*, 1979.

Plethora of Matching Semantics – Example

Example

A: Arts Book
B: Biography Book
C: Children's Book
M: Music CD



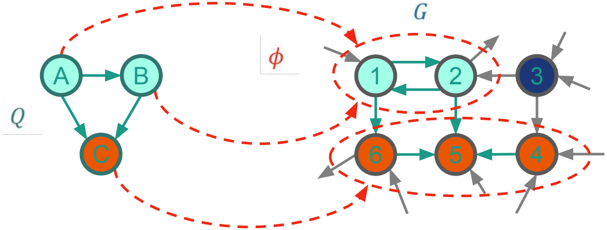
a) Q: Pattern   b) G: Data Graph

Results of Different Semantics

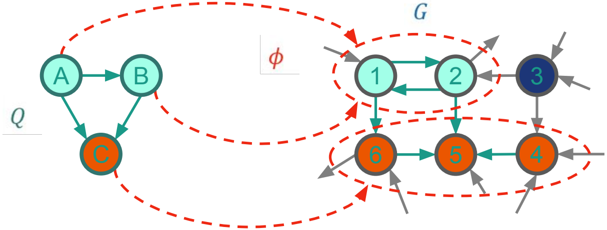
Model	Subgraph Results	Count
Graph Simulation	$\Phi(1,2,3,4) \rightarrow (\{1,6,8,12,16,19,20,24,27,30\}, \{2,7,13,15,17,21,23,26,29\}, \{3,4,5,9,11,14,18,22,25,28\}, \{3,4,5,9,11,14,18,22,25,28\})$	29
Dual Simulation	$\Phi(1,2,3,4) \rightarrow (\{1,6,8,12,16,19,20,24,27,30\}, \{2,7,13,15,17,21,23,26,29\}, \{3,4,5,9,14,18,22,25,28\}, \{3,4,5,9,14,18,22,25,28\})$	28
Strong Simulation	$\Phi(1,2,3,4) \rightarrow (\{1,6,8\}, \{2,7\}, \{3,4,5,9\}, \{3,4,5,9\}), (12,13,14,14), (\{16,19,20\}, \{15,17,21\}, \{14,18,22\}, \{14,18,22\})$	20
Strict Simulation	$\Phi(1,2,3,4) \rightarrow (\{1,6,8\}, \{2,7\}, \{3,4,5,9\}, \{3,4,5,9\}), (12,13,14,14)$	12
Tight Simulation	$\Phi(1,2,3,4) \rightarrow (1,2,\{3,4,5\},\{3,4,5\}), (12,13,14,14)$	8
CAR-Tight Simulation	$\Phi(1,2,3,4) \rightarrow (1,2,\{3,4,5\},\{3,4,5\})$	5
Graph Homomorphism	$f(1,2,3,4) \rightarrow (1,2,3,4), (1,2,3,5), (1,2,4,5), (1,2,3,3), (1,2,4,4), (1,2,5,5), (12,13,14,14)$	8
Subgraph Isomorphism	$f(1,2,3,4) \rightarrow (1,2,3,4), (1,2,3,5), (1,2,4,5)$	5

Simulation-Based Semantic

Graph Simulation



Graph Simulation



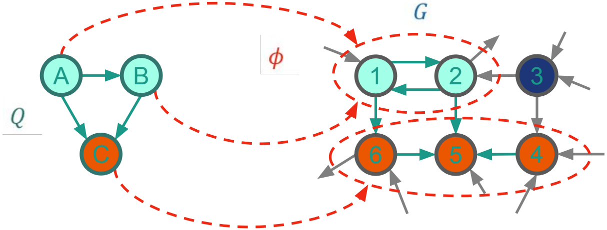
Definition

- Given a query graph $Q = (V_Q, E_Q)$ and a data graph $G = (V_G, E_G)$

Graph Simulation

Example

- $\phi(A) = \{1, 2\}$
- $\phi(B) = \{1, 2\}$
- $\phi(C) = \{6, 5, 4\}$



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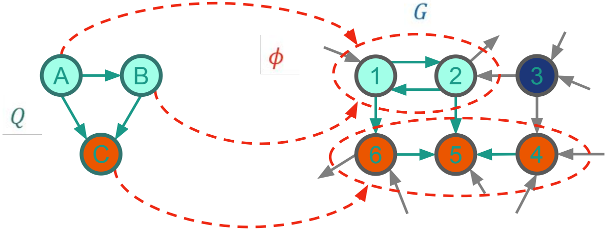

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- Given a query graph $Q = (V_Q, E_Q)$ and a data graph $G = (V_G, E_G)$
- A graph simulation matching is a function $\phi\colon V_Q \to \mathcal{P}(V_G)$ such that for all $v_Q \in V_Q$

Graph Simulation

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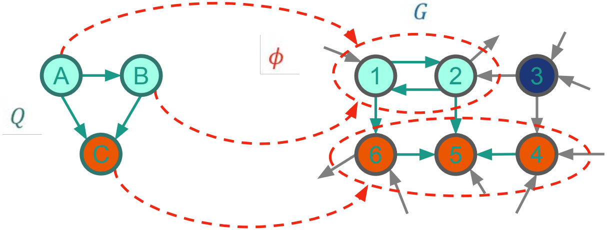

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    1. for all $v_G \in \phi(v_Q)$ the properties of v_G and v_Q match (here: they have the same color) and

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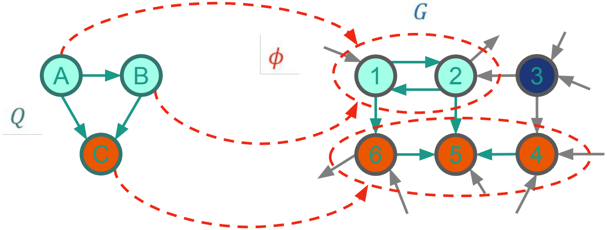

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  2. for all $v_G \in \phi(v_Q)$ and $(v_Q, w_Q) \in E_Q$ there is a $w_G \in \phi(w_Q)$ with $(v_G, w_G) \in E_G$

Graph Simulation

Example

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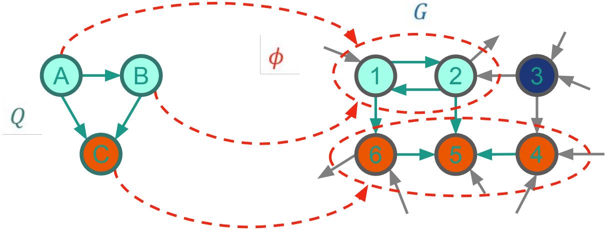

- $1 \in \phi(A)$ and $(A, B) \in E_Q$: there is $2 \in \phi(B)$ with $(1, 2) \in E_G$

Graph Simulation

Example

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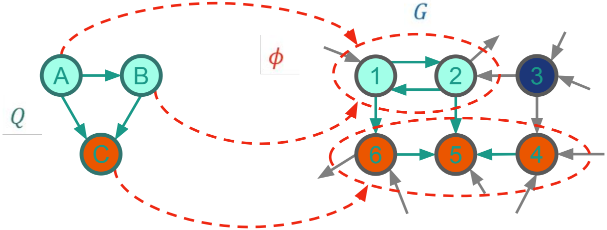

- $1 \in \phi(A)$ and $(A, B) \in E_Q$: there is $2 \in \phi(B)$ with $(1, 2) \in E_G$
- $1 \in \phi(A)$ and $(A, C) \in E_Q$: there is $6 \in \phi(C)$ with $(1, 6) \in E_G$

Graph Simulation

Example

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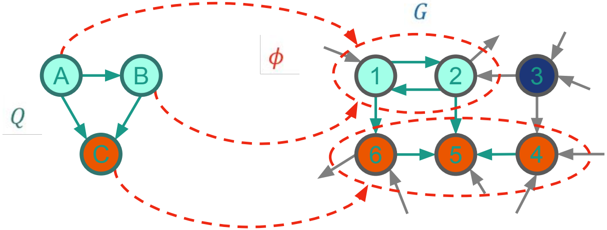

- $1 \in \phi(A)$ and $(A, B) \in E_Q$: there is $2 \in \phi(B)$ with $(1, 2) \in E_G$
- $1 \in \phi(A)$ and $(A, C) \in E_Q$: there is $6 \in \phi(C)$ with $(1, 6) \in E_G$
- $2 \in \phi(A)$ and $(A, B) \in E_Q$: there is $1 \in \phi(B)$ with $(2, 1) \in E_G$
- $2 \in \phi(A)$ and $(A, C) \in E_Q$: there is $5 \in \phi(C)$ with $(2, 5) \in E_G$

Graph Simulation

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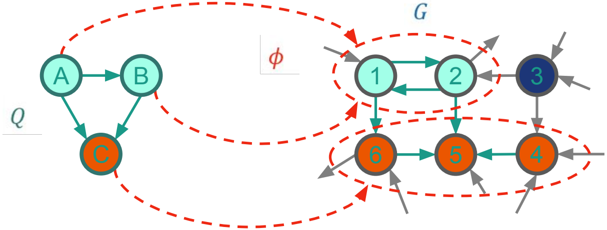

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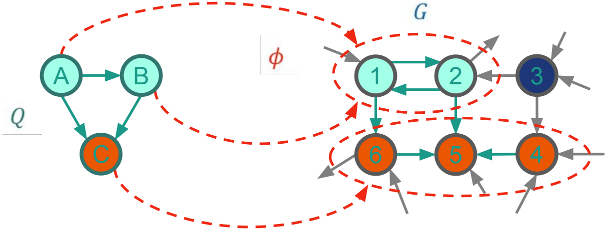

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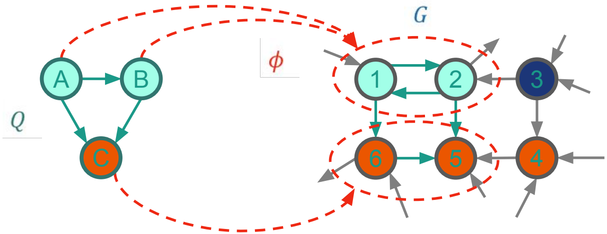
- $1 \in \phi(B)$ and $(B, C) \in E_Q$: there is $6 \in \phi(C)$ with $(1, 6) \in E_G$
- $2 \in \phi(B)$ and $(B, C) \in E_Q$: there is $5 \in \phi(C)$ with $(2, 5) \in E_G$
- C has no outgoing edges

Dual Simulation

Example

- $\phi(A) = \{1, 2\}$
- $\phi(B) = \{1, 2\}$
- $\phi(C) = \{6, 5\}$



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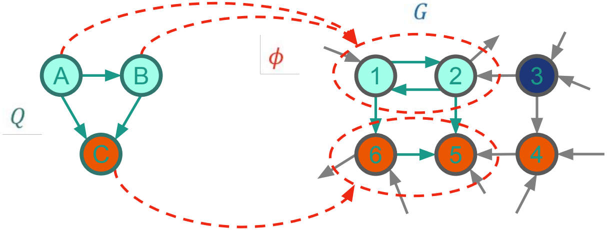
  3. for all $v_G \in \phi(v_Q)$ and $(w_Q, v_Q) \in E_Q$ there is a $w_G \in \phi(w_Q)$ with $(w_G, v_G) \in E_G$

Dual Simulation

Example

- $\phi(A) = \{1, 2\}$
- $\phi(B) = \{1, 2\}$
- $\phi(C) = \{6, 5\}$

- A has no incoming edges



Example

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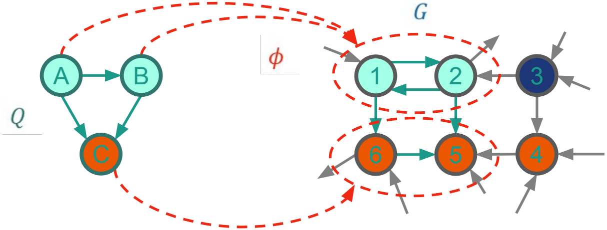
- $\phi(B) = \{1,2\}$
- $\phi(C) = \{6,5\}$

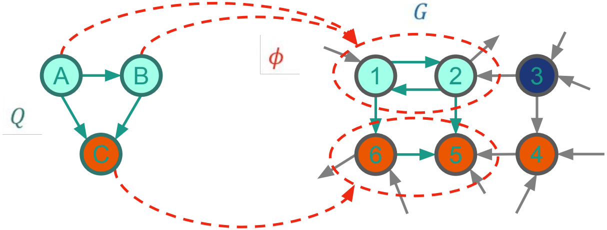


- A has no incoming edges
- $1 \in \phi(B)$ and $(A,B) \in E_Q$: there is $2 \in \phi(A)$ with $(2,1) \in E_G$

Dual Simulation

Example

- $\phi(A) = \{1, 2\}$
- $\phi(B) = \{1, 2\}$
- $\phi(C) = \{6, 5\}$



- A has no incoming edges
- $1 \in \phi(B)$ and $(A, B) \in E_Q$: there is $2 \in \phi(A)$ with $(2, 1) \in E_G$
- $2 \in \phi(B)$ and $(A, B) \in E_Q$: there is $1 \in \phi(A)$ with $(1, 2) \in E_G$

Example

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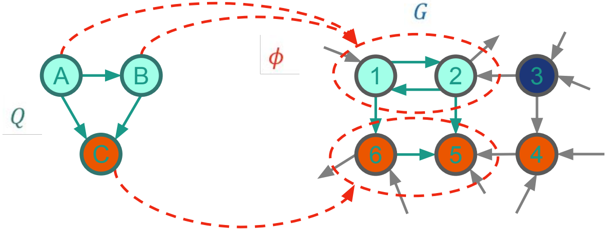

- A has no incoming edges
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- $2 \in \phi(B)$ and $(A, B) \in E_Q$: there is $1 \in \phi(A)$ with $(1, 2) \in E_G$
- $5 \in \phi(C)$ and $(A, C) \in E_Q$: there is $2 \in \phi(A)$ with $(2, 5) \in E_G$
- $5 \in \phi(C)$ and $(B, C) \in E_Q$: there is $2 \in \phi(B)$ with $(2, 5) \in E_G$

Dual Simulation

Example

- $\phi(A) = \{1, 2\}$
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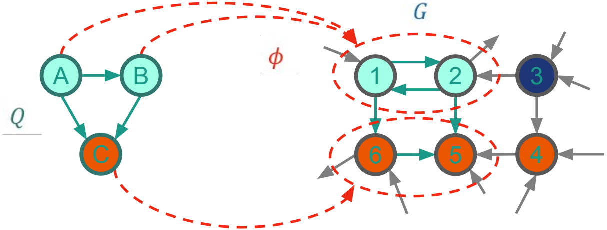

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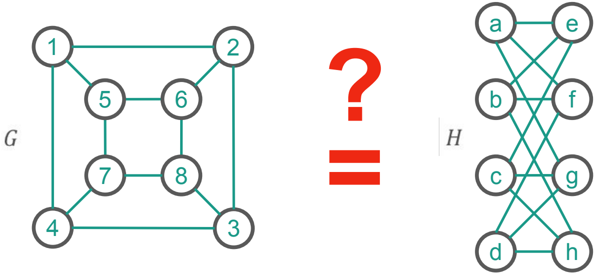
- $5 \in \phi(C)$ and $(A, C) \in E_Q$: there is $2 \in \phi(A)$ with $(2, 5) \in E_G$
- $5 \in \phi(C)$ and $(B, C) \in E_Q$: there is $2 \in \phi(B)$ with $(2, 5) \in E_G$
- $6 \in \phi(C)$ and $(A, C) \in E_Q$: there is $1 \in \phi(A)$ with $(1, 6) \in E_G$
- $6 \in \phi(C)$ and $(B, C) \in E_Q$: there is $1 \in \phi(B)$ with $(1, 6) \in E_G$

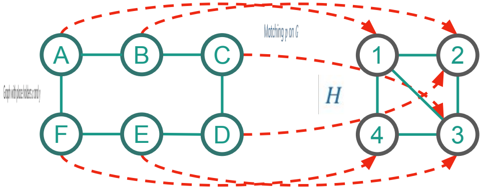
Morphism-Based Semantics

Graph Morphisms

ARE GRAPHS G AND H EQUAL/SIMILAR?



Graph Homomorphism



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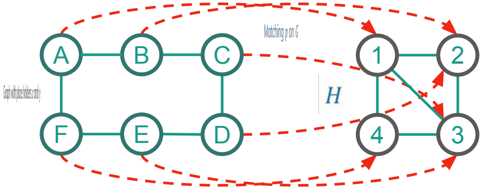

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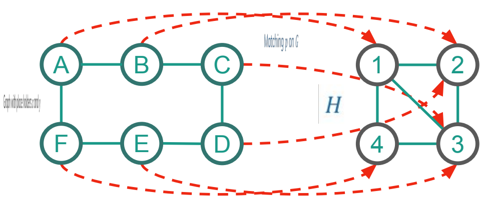

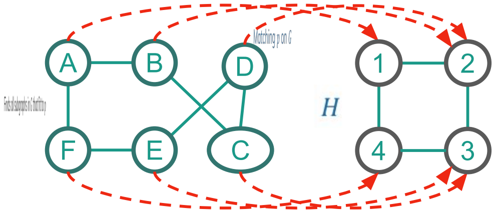

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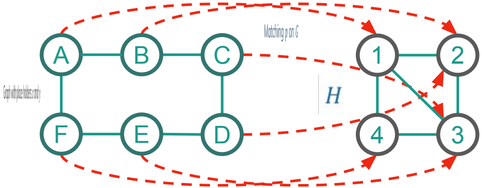

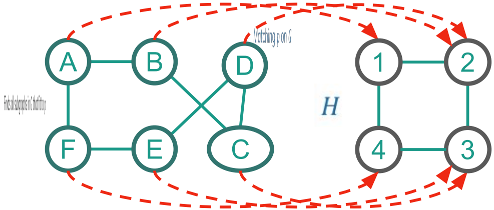

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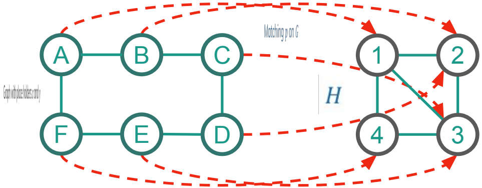

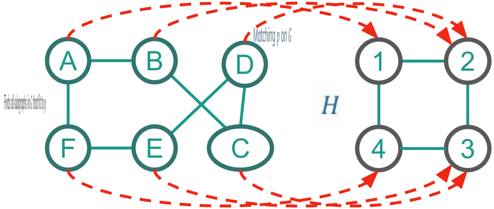

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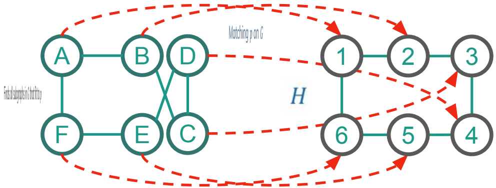

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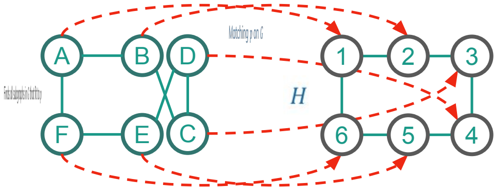

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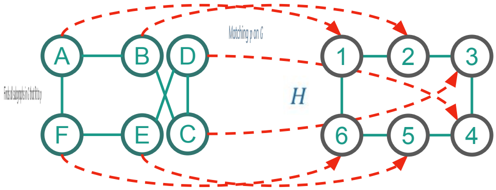

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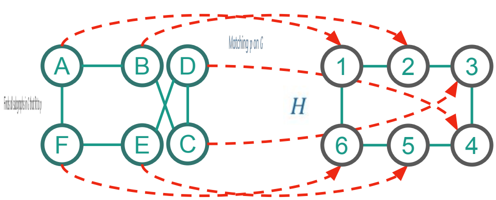

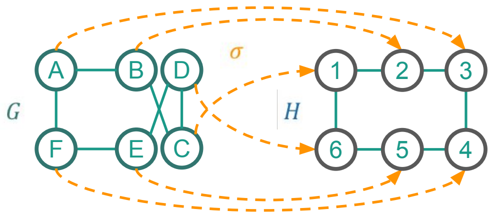

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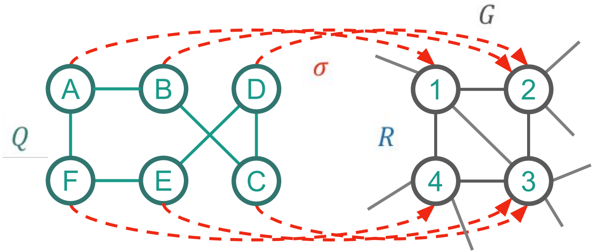
Cool!

Can we apply this to find subgraphs?

Subgraph Homomorphism Query



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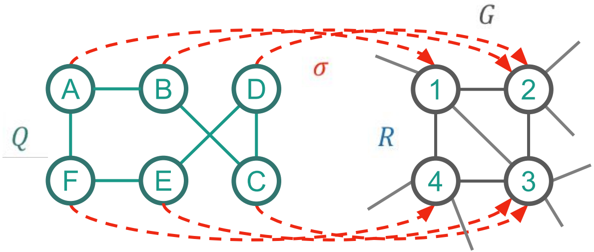

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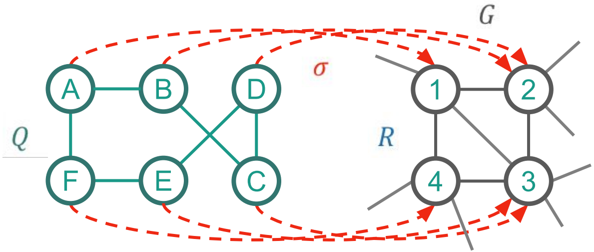

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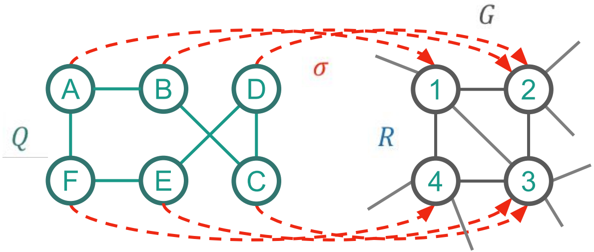

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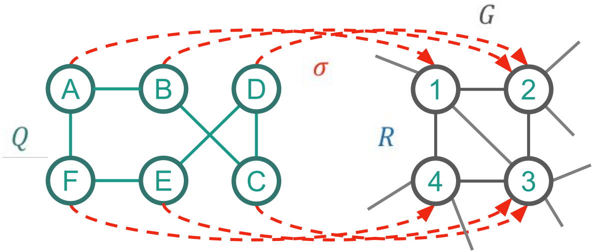

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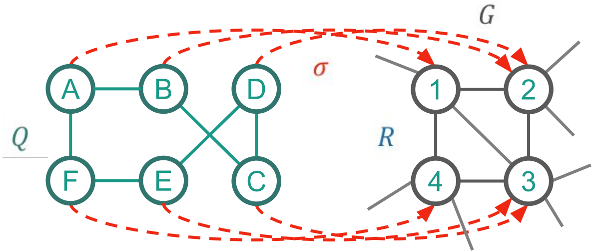

Notes

- Q can have more vertices or edges than R, i.e. in general $|V_Q| \geq |V_R|$ and $|E_Q| \geq |E_R|$
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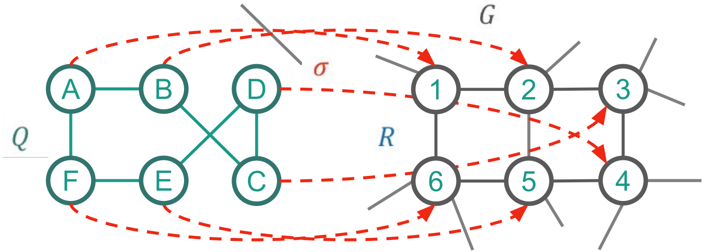

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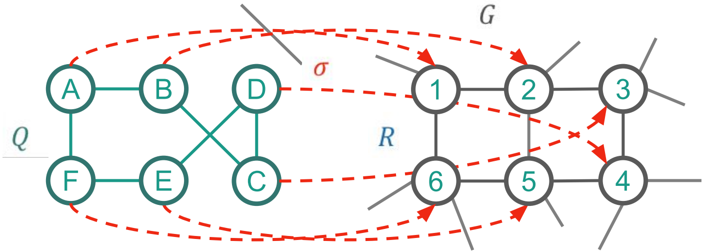

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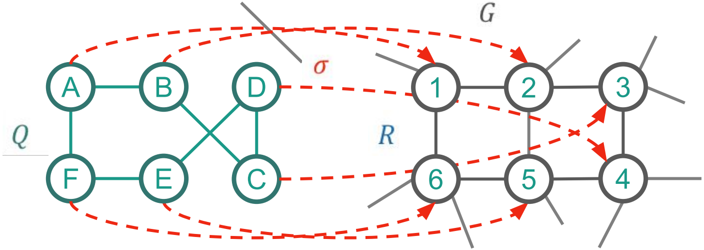

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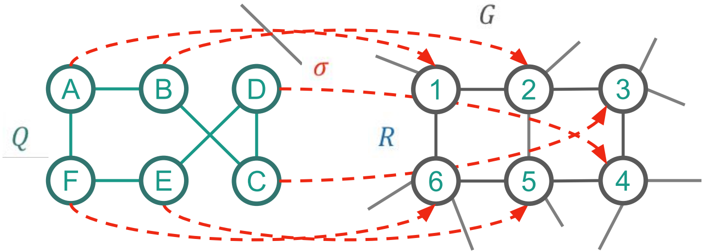

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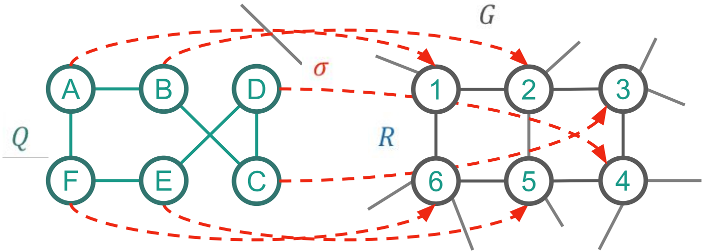

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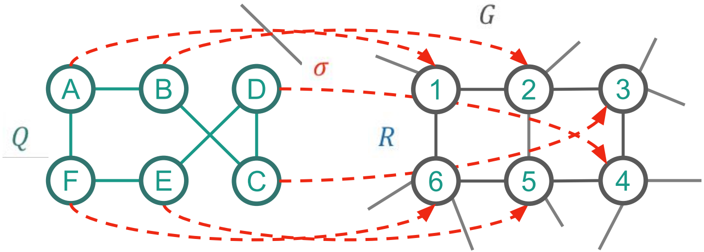

Notes

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Subgraph Matching Semantics

Subraph Homomorphism

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A single vertex in V_G can be matched multiple times in a homomorphic subgraph but only once in an isomorphic subgraph

Why Homomorphisms are useful...

Example

Look for all pairs of friends and the city each friend lives in

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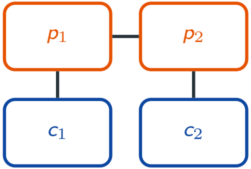
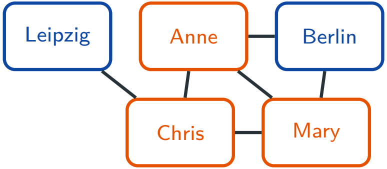
Look for all pairs of friends and the city each friend lives in

Query graph Q



Data graph G



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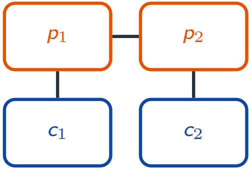
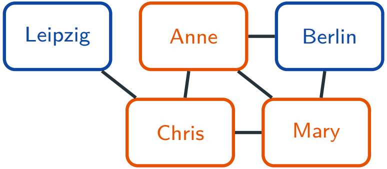


Data graph G



Assuming subgraph isomorphism, which tuples (c_1, p_1, p_2, c_2) represent results?

A: (Leipzig, Chris, Anne, Berlin)

B: (Leipzig, Chris, Mary, Berlin)

C: (Berlin, Anne, Mary, Berlin)

D: (Berlin, Anne, Chris, Leipzig)

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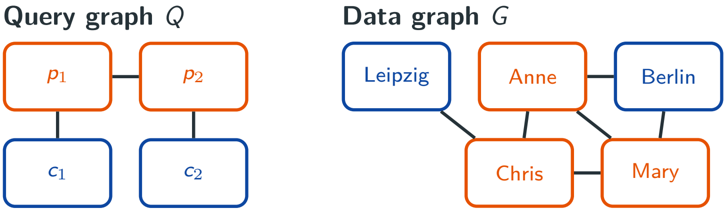

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- A: (Leipzig, Chris, Anne, Berlin)
- B: (Leipzig, Chris, Mary, Berlin)
- C: (Berlin, Anne, Mary, Berlin) no result: c_1 and c_2 cannot be matched to the same vertex "Berlin"
- D: (Berlin, Anne, Chris, Leipzig)

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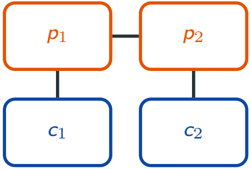
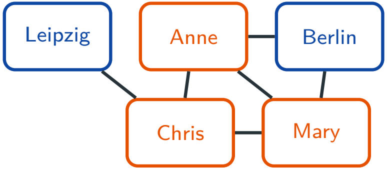
Query graph Q



Data graph G



Isomorphism finds only friends living in different cities

(Leipzig, Chris, Anne, Berlin), (Leipzig, Chris, Mary, Berlin), ...and permutations of these

Homomorphism additionally finds friends living in the same city

(Berlin, Mary, Anne, Berlin), (Berlin, Anne, Mary, Berlin)

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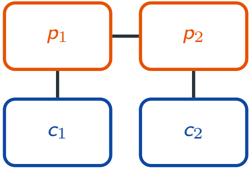
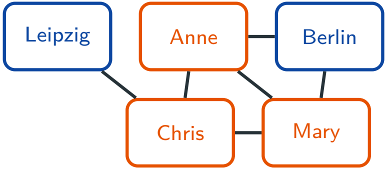
Query graph Q



Data graph G



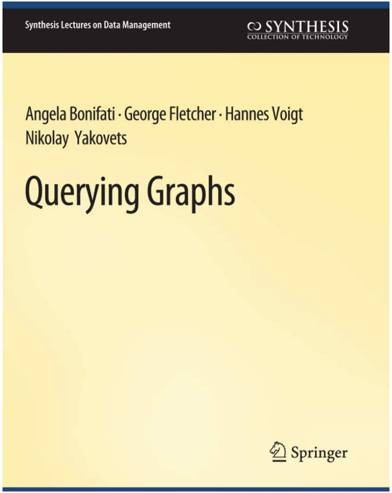
Isomorphism finds only friends living in different cities

(Leipzig, Chris, Anne, Berlin), (Leipzig, Chris, Mary, Berlin), ...and permutations of these

Homomorphism additionally finds friends living in the same city

(Berlin, Mary, Anne, Berlin), (Berlin, Anne, Mary, Berlin), (Berlin, Anne, Anne, Berlin), ...

Literature



Bonifati et al., *Querying Graphs*, 2018

References

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