

Big Graph Processing Systems

Part I: Graph Query Paradigms and their Semantics

► **Chapter 2:** Graph Query Language Classes

Christopher Spinrath

CNRS – LIRIS – Lyon 1 Université

DISS Master 2025

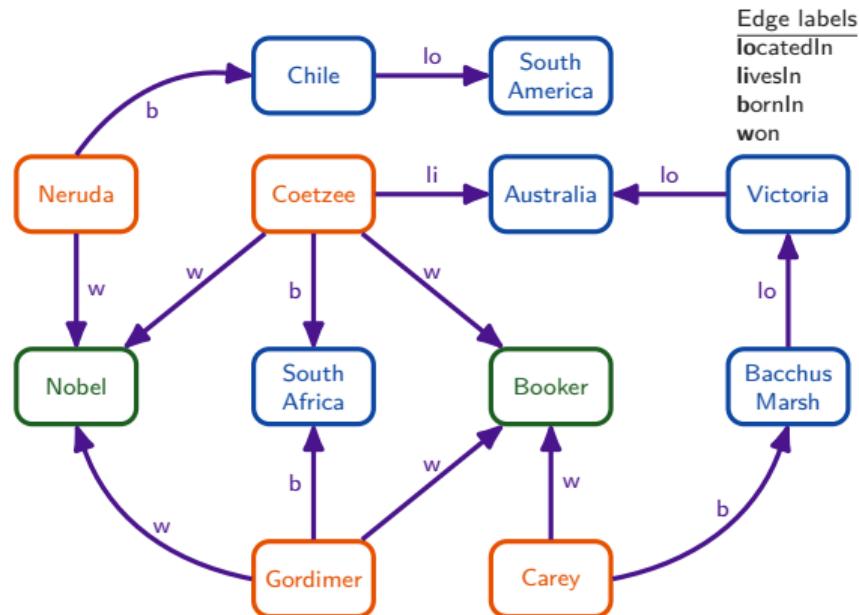
This presentation is an adaption of slides from Angela Bonifati



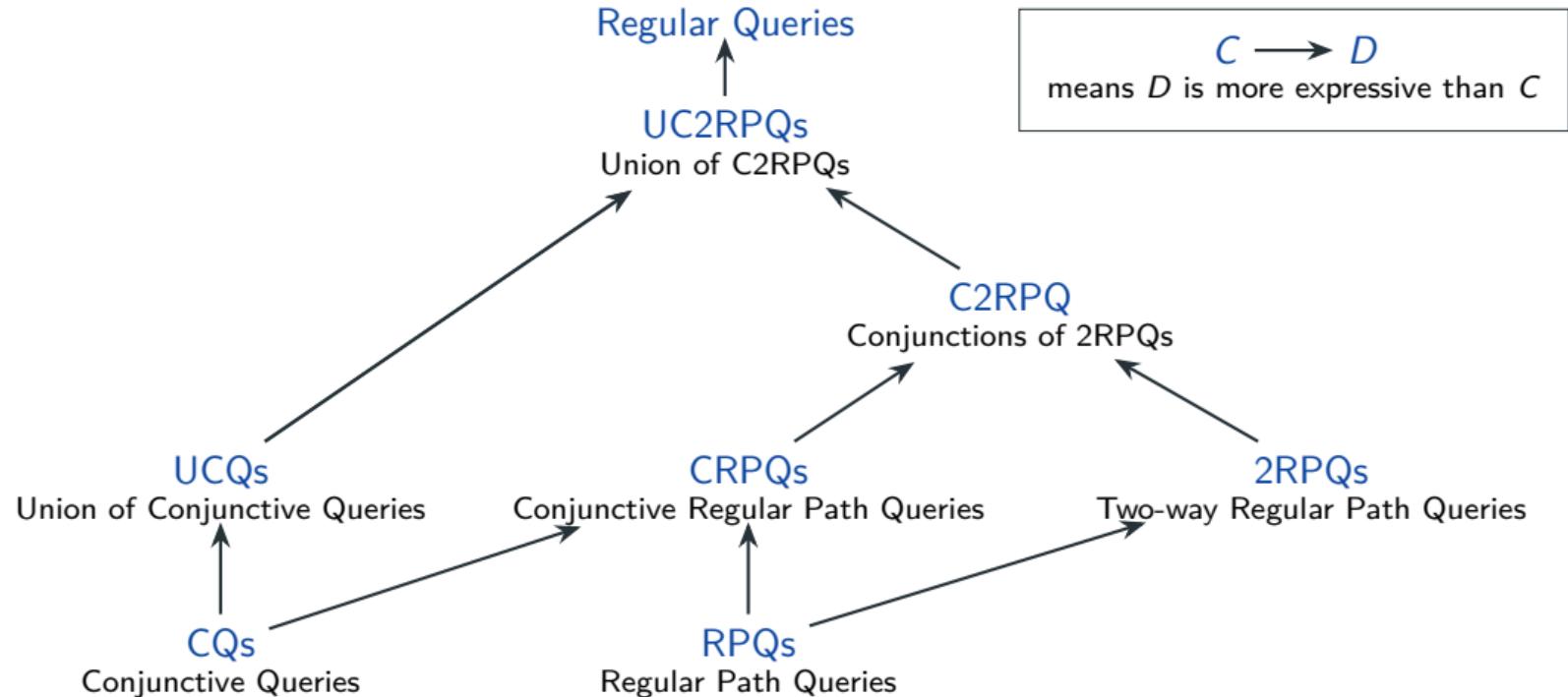
Graph Data Model

Graph data model for this chapter

- ▶ RDF-like data
- ▶ Directed graph with labeled edges
- ▶ $G = (V, E, \Sigma)$ where
 - ▶ V is the set of vertices,
 - ▶ $E \subseteq V \times \Sigma \times V$ is the set of labeled edges, and
 - ▶ Σ is the set (or alphabet) of labels.
- ▶ Node colors are for readability only



Query Language Classes – Overview



Conjunctive Queries (CQs) – The Idea

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Idea

- ▶ Querying for patterns
- ▶ A query is given as a set of **edge predicates** (x_i, a_i, y_i)
- ▶ x_i, y_i are **vertex variables** or **constants** and a_i is an edge label

Example

- ▶ $(x, \text{hasWon}, \text{Nobel})$
- ▶ $(x, \text{hasWon}, \text{Booker})$
- ▶ $(x, \text{bornIn}, \text{South Africa})$

Conjunctive Queries (CQs) – The Idea

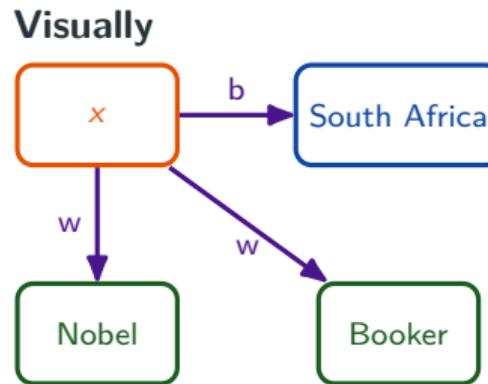
[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Idea

- ▶ Querying for patterns
- ▶ A query is given as a set of **edge predicates** (x_i, a_i, y_i)
- ▶ x_i, y_i are **vertex variables** or **constants** and a_i is an edge label

Example

- ▶ $(x, \text{hasWon}, \text{Nobel})$
- ▶ $(x, \text{hasWon}, \text{Booker})$
- ▶ $(x, \text{bornIn}, \text{South Africa})$



Conjunctive Queries (CQs)

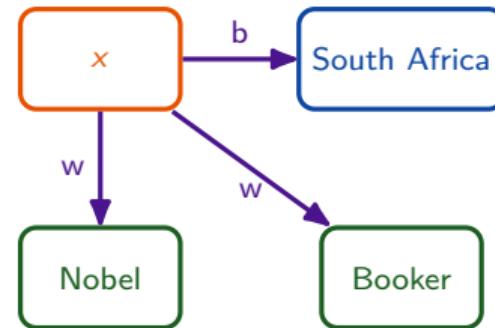
Syntax

- ▶ A *conjunctive query* Q is an expression

$$\text{ans}(z_1, \dots, z_n) \leftarrow \bigwedge_{1 \leq i \leq m} (x_i, a_i, y_i)$$

- ▶ Each x_i and each y_i is a vertex variable or a constant
- ▶ Each $a_i \in \Sigma$ is an edge label
- ▶ Each *free variable* is some x_i or y_i

Visually



Conjunctive Queries (CQs)

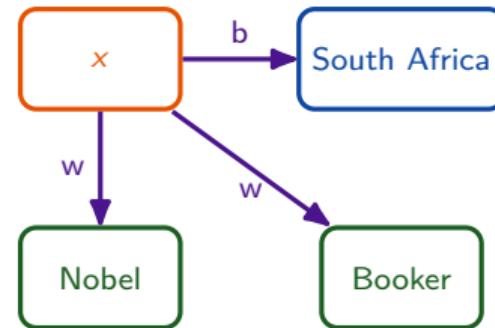
Syntax

- ▶ A *conjunctive query* Q is an expression

$$\text{ans}(z_1, \dots, z_n) \leftarrow \bigwedge_{1 \leq i \leq m} (x_i, a_i, y_i)$$

- ▶ Each x_i and each y_i is a vertex variable or a constant
- ▶ Each $a_i \in \Sigma$ is an edge label
- ▶ Each *free variable* is some x_i or y_i
- ▶ A conjunctive query is a formula in the \exists, \wedge -fragment of first-order logic

Visually



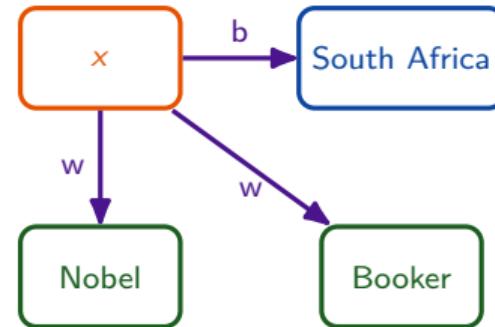
Conjunctive Queries (CQs)

Syntax

- ▶ A *conjunctive query* Q is an expression

$$\text{ans}(z_1, \dots, z_n) \leftarrow \bigwedge_{1 \leq i \leq m} (x_i, a_i, y_i)$$

Visually



Semantics

- ▶ Let $\sigma: X \rightarrow V$ be a variable binding, i.e. a mapping of variables to vertices of the graph G

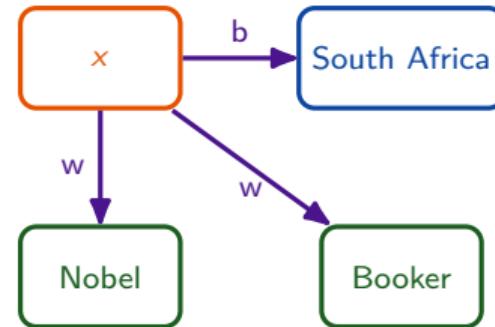
Conjunctive Queries (CQs)

Syntax

- ▶ A *conjunctive query* Q is an expression

$$\text{ans}(z_1, \dots, z_n) \leftarrow \bigwedge_{1 \leq i \leq m} (x_i, a_i, y_i)$$

Visually



Semantics

- ▶ Let $\sigma: X \rightarrow V$ be a variable binding, i.e. a mapping of variables to vertices of the graph G
- ▶ Say relation $(G, \sigma) \models Q$ holds iff $(\sigma(x_i), a_i, \sigma(y_i)) \in E$ for all $1 \leq i \leq m$

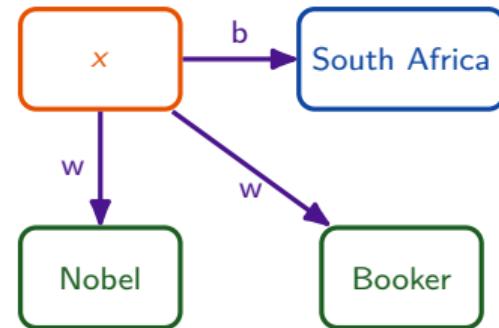
Conjunctive Queries (CQs)

Syntax

- ▶ A *conjunctive query* Q is an expression

$$\text{ans}(z_1, \dots, z_n) \leftarrow \bigwedge_{1 \leq i \leq m} (x_i, a_i, y_i)$$

Visually



Semantics

- ▶ Let $\sigma: X \rightarrow V$ be a variable binding, i.e. a mapping of variables to vertices of the graph G
- ▶ Say relation $(G, \sigma) \models Q$ holds iff $(\sigma(x_i), a_i, \sigma(y_i)) \in E$ for all $1 \leq i \leq m$
- ▶ Then the *query result* $Q(G)$ is the set of tuples $(\sigma(z_1), \dots, \sigma(z_n))$ such that $(G, \sigma) \models Q$:

$$Q(G) = \{(\sigma(z_1), \dots, \sigma(z_n)) \mid (G, \sigma) \models Q\}$$

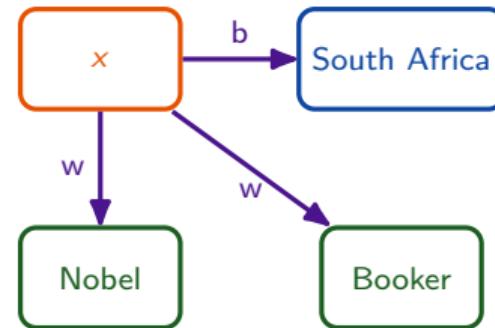
Conjunctive Queries (CQs)

Syntax

- ▶ A *conjunctive query* Q is an expression

$$\text{ans}(z_1, \dots, z_n) \leftarrow \bigwedge_{1 \leq i \leq m} (x_i, a_i, y_i)$$

Visually



Semantics

- ▶ Let $\sigma: X \rightarrow V$ be a variable binding, i.e. a mapping of variables to vertices of the graph G
- ▶ Say relation $(G, \sigma) \models Q$ holds iff $(\sigma(x_i), a_i, \sigma(y_i)) \in E$ for all $1 \leq i \leq m$
- ▶ Then the *query result* $Q(G)$ is the set of tuples $(\sigma(z_1), \dots, \sigma(z_n))$ such that $(G, \sigma) \models Q$:

$$Q(G) = \{(\sigma(z_1), \dots, \sigma(z_n)) \mid (G, \sigma) \models Q\}$$

Corresponds to
a homomorphism

Conjunctive Queries (CQs) – Example

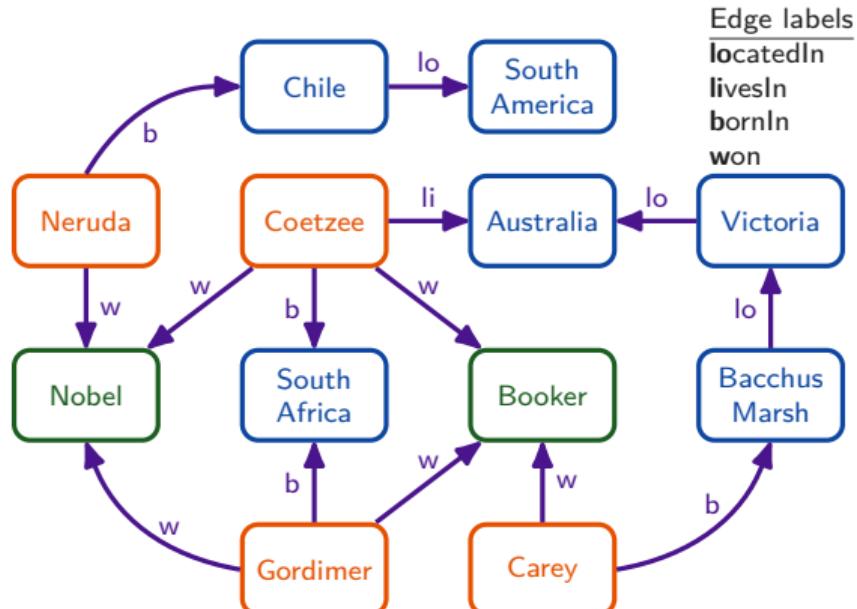
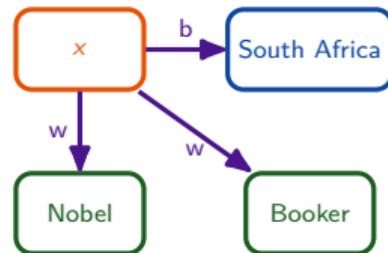
[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Example

All authors born in South Africa who have won both the Nobel and Booker prizes

```
ans(x) ← (x, hasWon, Nobel)
          ∧ (x, hasWon, Booker)
          ∧ (x, bornIn, South Africa)
```

Visually



Result?

Conjunctive Queries (CQs) – Example

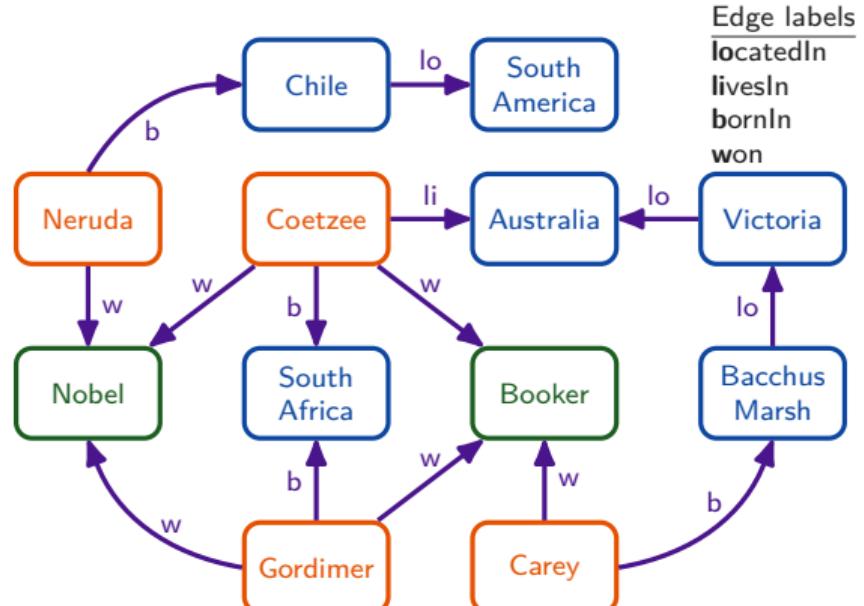
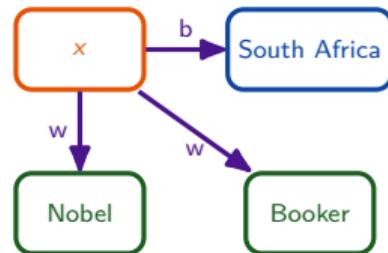
[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Example

All authors born in South Africa who have won both the Nobel and Booker prizes

```
ans(x) ← (x, hasWon, Nobel)  
      ∧ (x, hasWon, Booker)  
      ∧ (x, bornIn, South Africa)
```

Visually



Result?



Conjunctive Queries (CQs) – Example

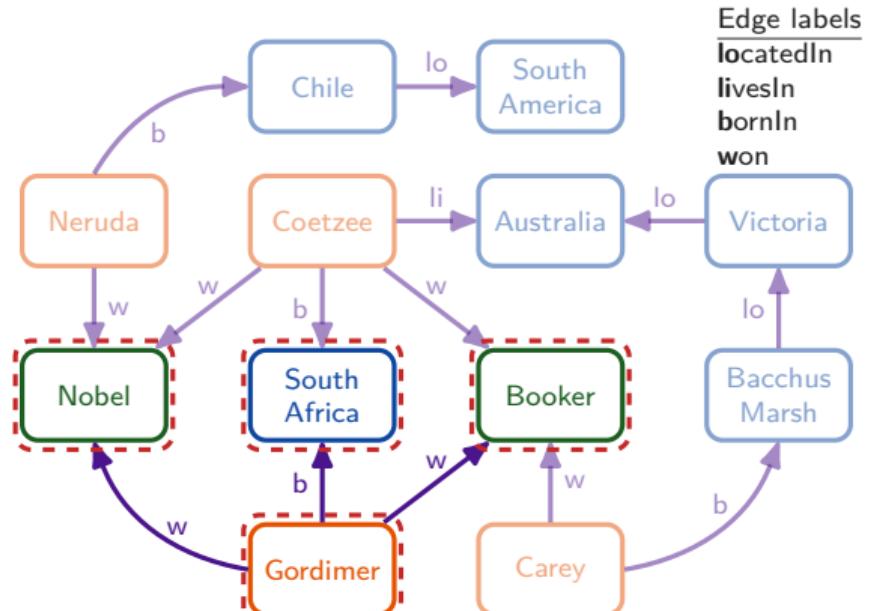
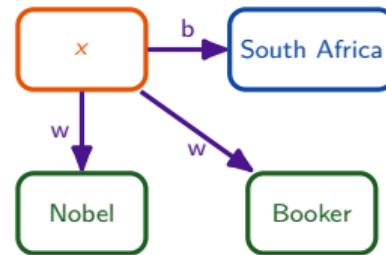
[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Example

All authors born in South Africa who have won both the Nobel and Booker prizes

```
ans(x) ← (x, hasWon, Nobel)  
      ∧ (x, hasWon, Booker)  
      ∧ (x, bornIn, South Africa)
```

Visually



Result?



Union of Conjunctive Queries (UCQs)

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Idea

Union of conjunctive queries with the same free variables

Union of Conjunctive Queries (UCQs)

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Idea

Union of conjunctive queries with the same free variables

Definition

A **union of conjunctive queries** Q is an expression

$$\text{ans}(z_1, \dots, z_n) \leftarrow \bigcup_{1 \leq j \leq k} Q_j(z_1, \dots, z_n)$$

where each Q_j is a conjunctive query with free variables z_1, \dots, z_n

Union of Conjunctive Queries (UCQs)

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Idea

Union of conjunctive queries with the same free variables

Definition

A **union of conjunctive queries** Q is an expression

$$\text{ans}(z_1, \dots, z_n) \leftarrow \bigcup_{1 \leq j \leq k} Q_j(z_1, \dots, z_n)$$

where each Q_j is a conjunctive query with free variables z_1, \dots, z_n

Semantics

The **query result** $Q(G)$ is the union $Q(G) = \bigcup_{1 \leq j \leq k} Q_j(G)$

Regular Path Queries (RPQs)

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Idea

- ▶ Querying for reachability
- ▶ A query is given as a **path predicate** consisting of
 - ▶ a pair of vertex variables x, y , and
 - ▶ a path expression r
- ▶ A **path expression** is a regular expression over edge labels
- ▶ A pair of vertices is in the **query result** if and only if they are connected by a path conforming to the path expression

Example

$\text{ans}(x, y) \leftarrow (x, (\text{bornIn} \mid \text{livesIn}) \cdot \text{locatedIn}^*, y)$

Regular Path Queries (RPQs) – Syntax

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Syntax

- ▶ A **regular path query** Q is an expression

$$\text{ans}(x, y) \leftarrow (x, r, y)$$

- ▶ where x and y are vertex variables, and
- ▶ r is a **regular expression** over the alphabet Σ of edge labels
- ▶ Regular expressions over Σ are all and only those expressions recursively generated as follows:

Regular Path Queries (RPQs) – Syntax

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Syntax

- ▶ A **regular path query** Q is an expression

$$\text{ans}(x, y) \leftarrow (x, r, y)$$

- ▶ where x and y are vertex variables, and
- ▶ r is a **regular expression** over the alphabet Σ of edge labels
- ▶ Regular expressions over Σ are all and only those expressions recursively generated as follows:
 - ▶ If $a \in \Sigma$, then a is a regular expression
 - ▶ If r_1 and r_2 are regular expressions then
 - ▶ $r_1 \cdot r_2$ (concatenation),
 - ▶ $r_1 | r_2$ (disjunction), and
 - ▶ $(r_1)^\star$ (Kleene star)

are regular expressions

Regular Path Queries (RPQs) – Syntax

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Syntax

- ▶ A **regular path query** Q is an expression

$$\text{ans}(x, y) \leftarrow (x, r, y)$$

- ▶ where x and y are vertex variables, and
- ▶ r is a **regular expression** over the alphabet Σ of edge labels
- ▶ Regular expressions over Σ are all and only those expressions recursively generated as follows:
 - ▶ If $a \in \Sigma$, then a is a regular expression
 - ▶ If r_1 and r_2 are regular expressions then
 - ▶ $r_1 \cdot r_2$ (concatenation),
 - ▶ $r_1 | r_2$ (disjunction), and
 - ▶ $(r_1)^\star$ (Kleene star)
- ▶ are regular expressions
- ▶ Concatenation takes precedence, unnecessary parentheses are omitted

Regular Path Queries (RPQs) – Semantics

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Semantics

- ▶ Given a RPQ $Q = \text{ans}(x, y) \leftarrow (x, r, y)$ and a data graph G
- ▶ The **query result** $Q(G)$ of Q in G is the set $r(G)$ recursively defined as follows:

Regular Path Queries (RPQs) – Semantics

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Semantics

- ▶ Given a RPQ $Q = \text{ans}(x, y) \leftarrow (x, r, y)$ and a data graph G
- ▶ The **query result** $Q(G)$ of Q in G is the set $r(G)$ recursively defined as follows:
 - ▶ If $r = a \in \Sigma$, then $r(G) = \{(s, t) \mid (s, a, t) \in E\}$

Regular Path Queries (RPQs) – Semantics

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Semantics

- ▶ Given a RPQ $Q = \text{ans}(x, y) \leftarrow (x, r, y)$ and a data graph G
- ▶ The **query result** $Q(G)$ of Q in G is the set $r(G)$ recursively defined as follows:
 - ▶ If $r = a \in \Sigma$, then $r(G) = \{(s, t) \mid (s, a, t) \in E\}$
 - ▶ If $r = r_1 \cdot r_2$, then

$$r(G) = \{(s, t) \mid \text{there is } z \in V \text{ with } (s, z) \in r_1(G) \text{ and } (z, t) \in r_2(G)\}$$

Regular Path Queries (RPQs) – Semantics

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Semantics

- ▶ Given a RPQ $Q = \text{ans}(x, y) \leftarrow (x, r, y)$ and a data graph G
- ▶ The **query result** $Q(G)$ of Q in G is the set $r(G)$ recursively defined as follows:
 - ▶ If $r = a \in \Sigma$, then $r(G) = \{(s, t) \mid (s, a, t) \in E\}$
 - ▶ If $r = r_1 \cdot r_2$, then

$$r(G) = \{(s, t) \mid \text{there is } z \in V \text{ with } (s, z) \in r_1(G) \text{ and } (z, t) \in r_2(G)\}$$

- ▶ If $r = r_1|r_2$, then $r(G) = r_1(G) \cup r_2(G)$

Regular Path Queries (RPQs) – Semantics

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Semantics

- ▶ Given a RPQ $Q = \text{ans}(x, y) \leftarrow (x, r, y)$ and a data graph G
- ▶ The **query result** $Q(G)$ of Q in G is the set $r(G)$ recursively defined as follows:
 - ▶ If $r = a \in \Sigma$, then $r(G) = \{(s, t) \mid (s, a, t) \in E\}$
 - ▶ If $r = r_1 \cdot r_2$, then

$$r(G) = \{(s, t) \mid \text{there is } z \in V \text{ with } (s, z) \in r_1(G) \text{ and } (z, t) \in r_2(G)\}$$

- ▶ If $r = r_1|r_2$, then $r(G) = r_1(G) \cup r_2(G)$
- ▶ If $r = (r_1)^*$, then

$$r(G) = \text{TC}(r_1(G)) \cup \{(s, s) \mid s \in V\}$$

where $\text{TC}(R)$ denotes the transitive closure of binary relation R

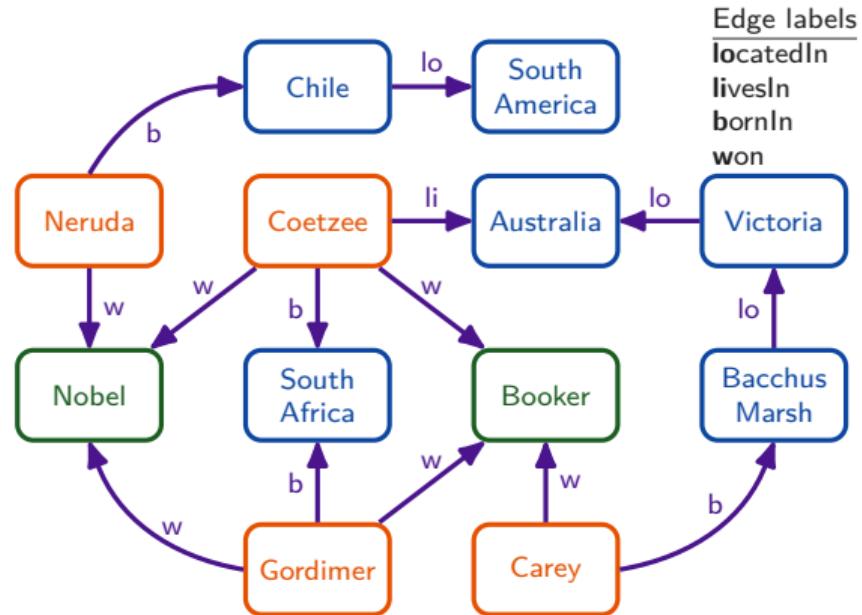
Regular Path Queries (RPQs) – Example

[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Example

All authors and where they live in or are born

$$\text{ans}(x, y) \leftarrow (x, (b|li) \cdot lo^*, y)$$



Regular Path Queries (RPQs) – Example

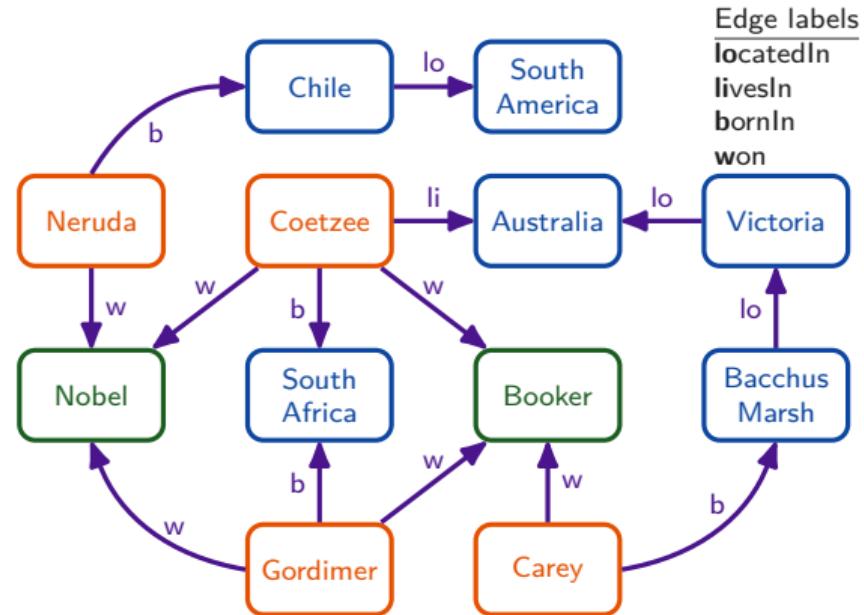
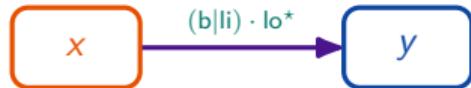
[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Example

All authors and where they live in or are born

$$\text{ans}(x, y) \leftarrow (x, (b|li) \cdot lo^*, y)$$

Visually



Regular Path Queries (RPQs) – Example

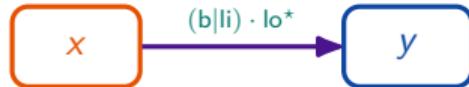
[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Example

All authors and where they live in or are born

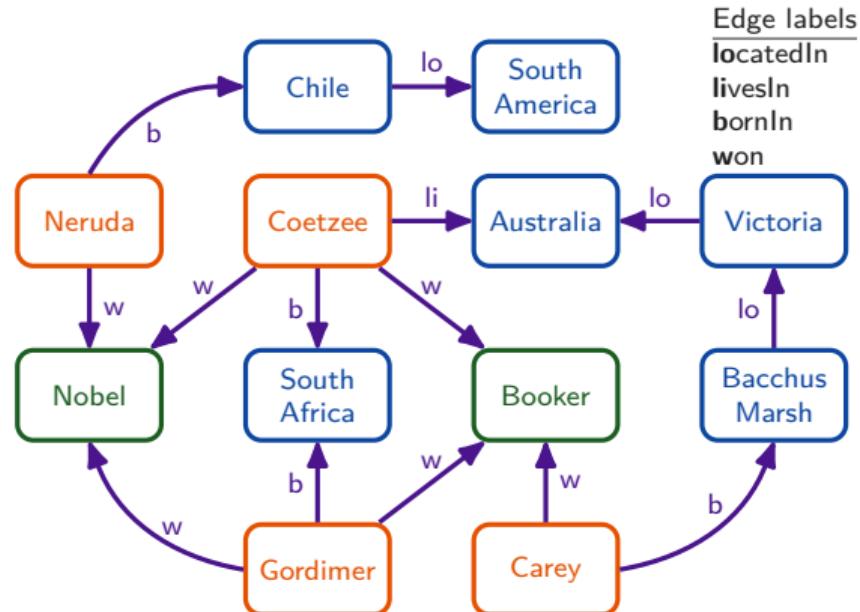
$\text{ans}(x, y) \leftarrow (x, (\mathbf{b}|\text{li}) \cdot \text{lo}^*, y)$

Visually



How many results?

A: 2 B: 8 C: 3 D: 6



Regular Path Queries (RPQs) – Example

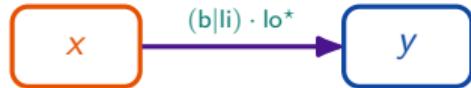
[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Example

All authors and where they live in or are born

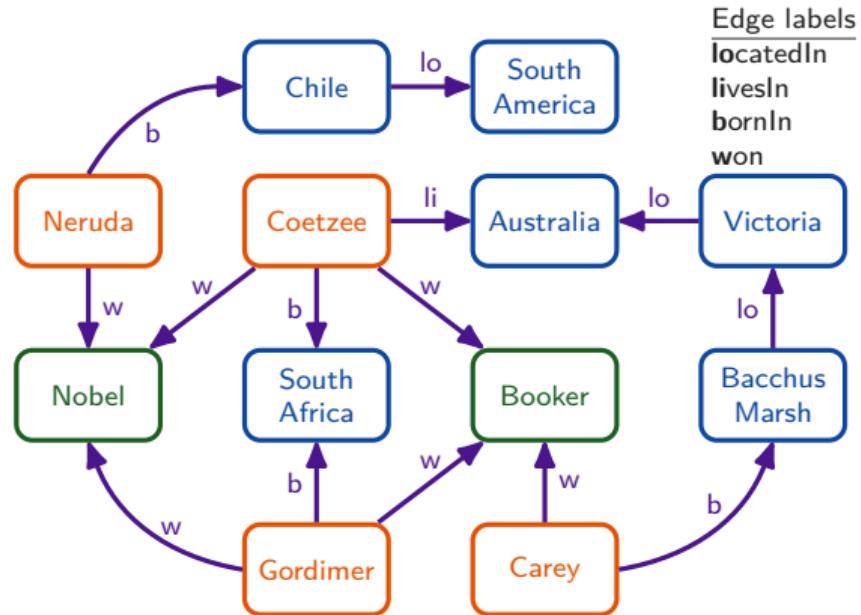
$$\text{ans}(x, y) \leftarrow (x, (b|li) \cdot lo^*, y)$$

Visually



8 Results

Neruda	South America	Carey	Australia
Coetzee	Australia	Carey	Victoria
Coetzee	South Africa	Carey	Bacchus Marsh
Gordimer	South Africa	Neruda	Chile



Two-Way Regular Path Queries (2RPQs)

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Like RPQs but with the following extension

Syntax

- ▶ Regular expressions over Σ are all and only those expressions recursively generated as follows:

Two-Way Regular Path Queries (2RPQs)

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Like RPQs but with the following extension

Syntax

- ▶ Regular expressions over Σ are all and only those expressions recursively generated as follows:
 - ▶ If $a \in \Sigma$, then a is a regular expression
 - ▶ If $a \in \Sigma$, then a^{-1} is a regular expression (reverse traversal)
 - ▶ If r_1 and r_2 are regular expressions then $r_1 \cdot r_2$, $r_1|r_2$, and $(r_1)^*$ are regular expressions

Two-Way Regular Path Queries (2RPQs)

[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Like RPQs but with the following extension

Syntax

- ▶ Regular expressions over Σ are all and only those expressions recursively generated as follows:
 - ▶ If $a \in \Sigma$, then a is a regular expression
 - ▶ If $a \in \Sigma$, then a^{-1} is a regular expression (reverse traversal)
 - ▶ If r_1 and r_2 are regular expressions then $r_1 \cdot r_2$, $r_1|r_2$, and $(r_1)^*$ are regular expressions

Semantics

- ▶ The query result $Q(G)$ of Q in G is the set $r(G)$ recursively defined as follows:
 - ▶ If $r = a \in \Sigma$, then $r(G) = \{(s, t) \mid (s, a, t) \in E\}$
 - ▶ If $r = a^{-1}$, $a \in \Sigma$, then $r(G) = \{(s, t) \mid (t, a, s) \in E\}$
 - ▶ If $r = r_1 \cdot r_2$, then $r(G) = \{(s, t) \mid \text{there is } z \in V \text{ with } (s, z) \in r_1(G) \text{ and } (z, t) \in r_2(G)\}$
 - ▶ If $r = r_1|r_2$, then $r(G) = r_1(G) \cup r_2(G)$
 - ▶ If $r = (r_1)^*$, then $r(G) = \text{TC}(r_1(G)) \cup \{(s, s) \mid s \in V\}$ where $\text{TC}(R)$ denotes the transitive closure of R

Conjunctive Regular Path Queries (CRPQs)

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Idea

- ▶ Querying for patterns including variable length paths
- ▶ Conjunctions of RPQs

Conjunctive Regular Path Queries (CRPQs)

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Idea

- ▶ Querying for patterns including variable length paths
- ▶ Conjunctions of RPQs

Syntax

- ▶ A **conjunctive regular path query** Q is an expression

$$\text{ans}(z_1, \dots, z_n) \leftarrow \bigwedge_{1 \leq i \leq m} (x_i, r_i, y_i)$$

- ▶ The x_i, y_i are vertex variables or constants
- ▶ Each r_i is a **regular expression** over the label alphabet Σ

Conjunctive Regular Path Queries (CRPQs)

[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Idea

- ▶ Querying for patterns including variable length paths
- ▶ Conjunctions of RPQs

Syntax

- ▶ A **conjunctive regular path query** Q is an expression

$$\text{ans}(z_1, \dots, z_n) \leftarrow \bigwedge_{1 \leq i \leq m} (x_i, r_i, y_i)$$

- ▶ The x_i, y_i are vertex variables or constants
- ▶ Each r_i is a **regular expression** over the label alphabet Σ

Semantics

- ▶ Let $\sigma: X \rightarrow V$ be a variable binding
- ▶ Say relation $(G, \sigma) \models Q$ holds if and only if $(\sigma(x_i), \sigma(y_i)) \in r_i(G)$ for all $1 \leq i \leq m$
- ▶ Then the **query result** $Q(G)$ is the set of all tuples $(\sigma(z_1), \dots, \sigma(z_n))$ such that $(G, \sigma) \models Q$

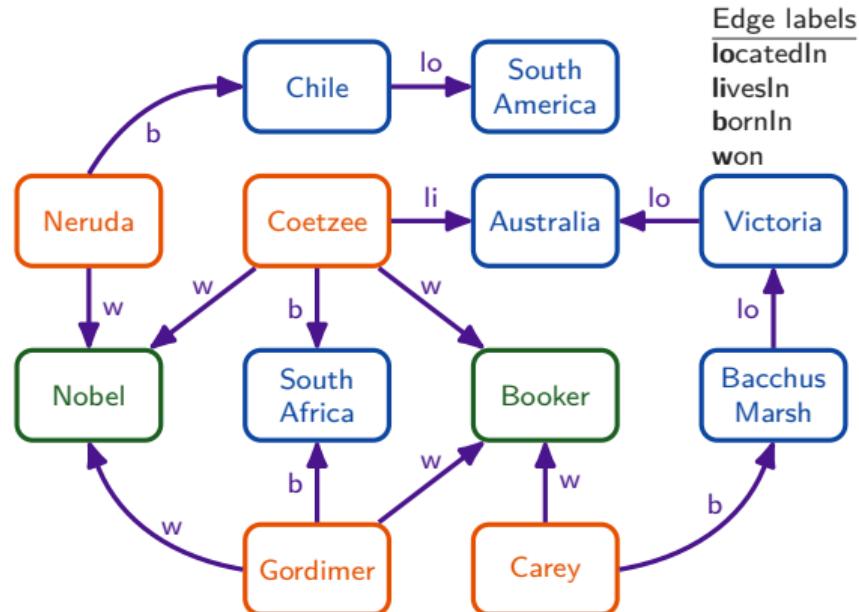
Conjunctive Regular Path Queries (CRPQs) – Example

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

Example

All Nobel+Booker winners and where they live

```
ans(x, y) ← (x, li · lo★, y)
           ∧ (x, w, Booker) ∧ (x, w, Nobel)
```



Conjunctive Regular Path Queries (CRPQs) – Example

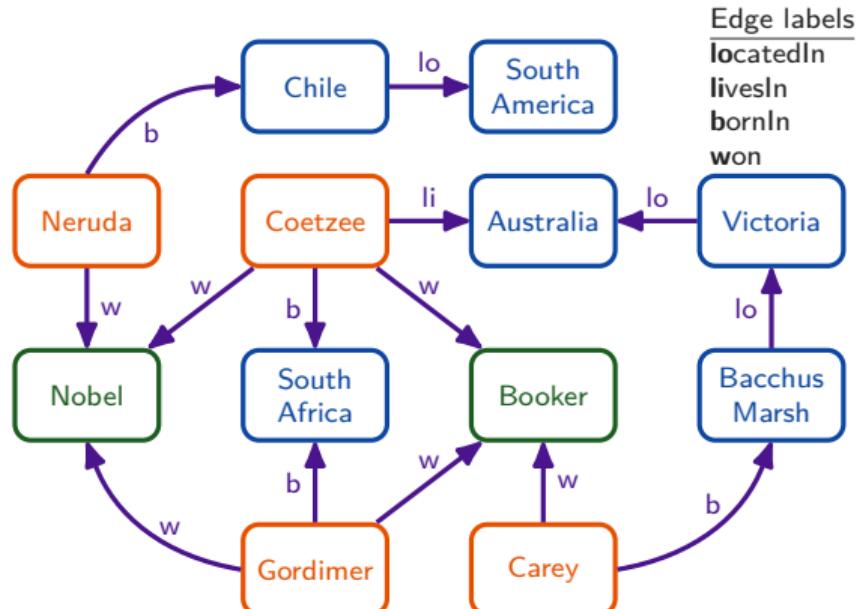
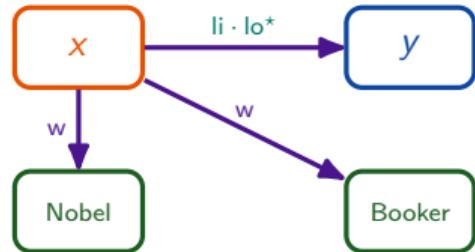
[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Example

All Nobel+Booker winners and where they live

$$\begin{aligned} \text{ans}(x, y) \leftarrow & (x, \text{li} \cdot \text{lo}^*, y) \\ \wedge & (x, w, \text{Booker}) \wedge (x, w, \text{Nobel}) \end{aligned}$$

Visually



Conjunctive Regular Path Queries (CRPQs) – Example

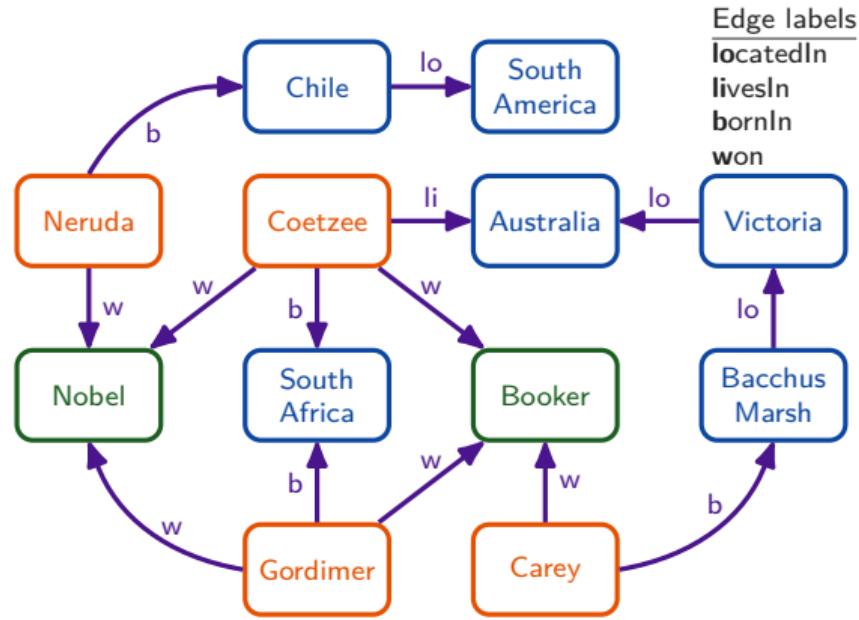
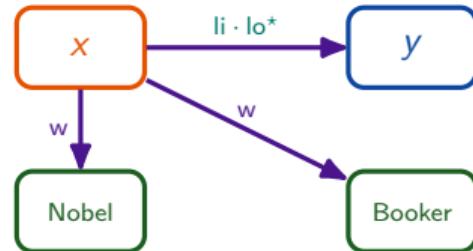
[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Example

All Nobel+Booker winners and where they live

$$\begin{aligned} \text{ans}(x, y) \leftarrow (x, \text{li} \cdot \text{lo}^*, y) \\ \wedge (x, w, \text{Booker}) \wedge (x, w, \text{Nobel}) \end{aligned}$$

Visually



Result?

A: (Gordimer, Australia) B: (Coetzee, Australia) C: (Coetzee, Bacchus Ma.) D: (Neruda, South Am.)

Conjunctive Regular Path Queries (CRPQs) – Example

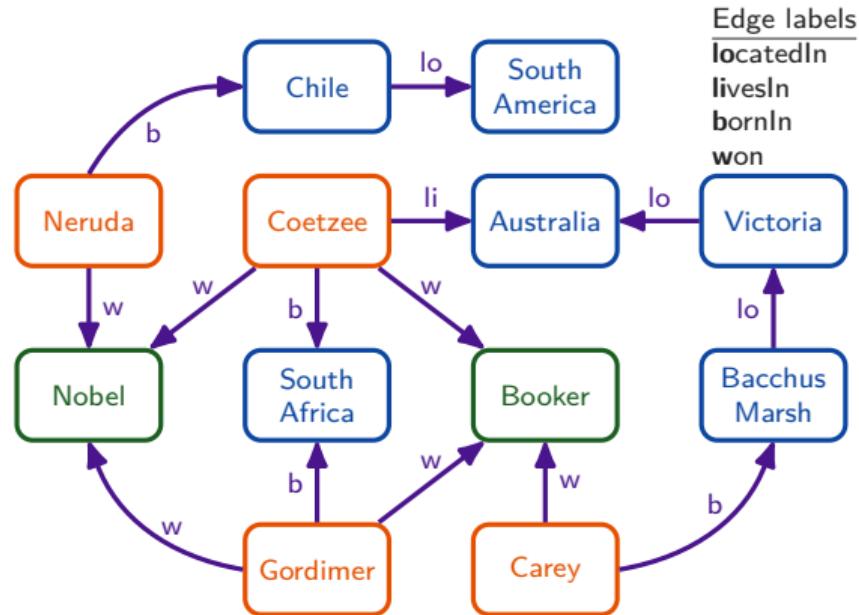
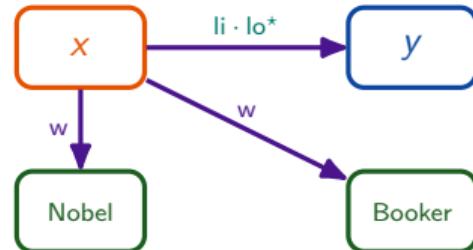
[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Example

All Nobel+Booker winners and where they live

$$\begin{aligned} \text{ans}(x, y) \leftarrow (x, \text{li} \cdot \text{lo}^*, y) \\ \wedge (x, w, \text{Booker}) \wedge (x, w, \text{Nobel}) \end{aligned}$$

Visually



Result?

A: (Gordimer, Australia) B: (Coetzee, Australia) C: (Coetzee, Bacchus Ma.) D: (Neruda, South Am.)

C2RPQs and UC2RPQs

[Wood, "Query languages for graph databases", *SIGMOD Rec.*, 2012]

C2RPQs

- ▶ Conjunctions of 2RPQs

UC2RPQs

- ▶ Unions of C2RPQs
- ▶ Class which most practical query languages (more or less) fall into

Example

$$\begin{aligned} \text{ans}(x, y) \leftarrow & (x, \text{actedIn} \cdot \text{actedIn}^{-1} \cdot (\text{actedIn} \cdot \text{actedIn}^{-1})^*, y) \\ & \wedge (x, \text{livesIn}, z) \wedge (y, \text{livesIn}, z) \end{aligned}$$

U $\text{ans}(x, y) \leftarrow (x, \text{worksFor} \cdot \text{partOf}^*, w) \wedge (y, \text{worksFor} \cdot \text{partOf}^*, w)$

[Wood, "Query languages for graph databases", SIGMOD Rec., 2012]

Regular Queries

- ▶ UC2RPQs can be **nested**, if they define a binary relation (that is, a set of edges)
- ▶ The left hand sides (heads) of other, binary UC2RPQs can be used as any other label
 - ▶ In particular, as part of regular expressions

Example

$$\begin{aligned} \text{ans}(x, c_x, y, c_y) \leftarrow & (x, \text{livesIn}, c_x) \wedge (x, \text{livesIn}, c_x) \\ & \wedge (c_x, \text{partnerCities}, c_y) \wedge (x, \text{friendsWithMutualFriend}^*, y) \end{aligned}$$
$$\text{friendsWithMutualFriend}(x, y) \leftarrow (x, \text{friend}, y) \wedge (x, \text{friend}, z) \wedge (y, \text{friend}, z)$$

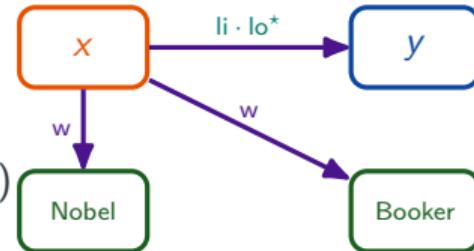
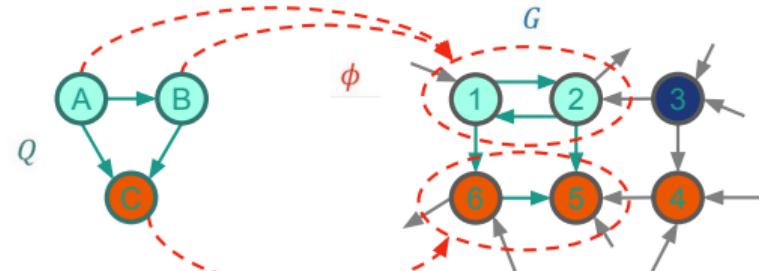
Summary

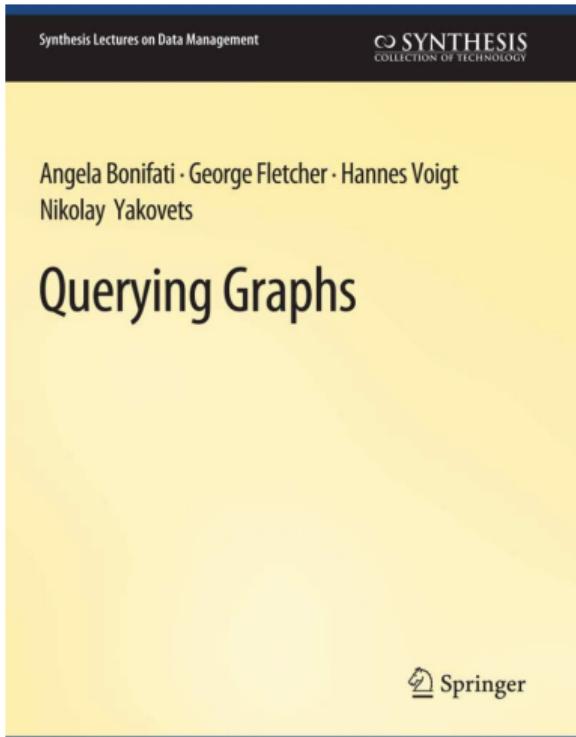
Matching Semantics

- ▶ Graph simulation
- ▶ Dual simulation
- ▶ Subgraph isomorphism
- ▶ Subgraph homomorphism

Query Types

- ▶ Conjunctive Queries (CQs)
- ▶ Regular Path Queries (RPQs)
- ▶ Conjunctive Regular Path Queries (CRPQs)
- ▶ Union of Conjunctive Two-Way Regular Path Queries (UC2RPQs)
- ▶ Regular Queries





Bonifati et al., *Querying Graphs*, 2018

References

-  Bonifati, Angela, George H. L. Fletcher, Hannes Voigt, and Nikolay Yakovets (2018). *Querying Graphs*. Synthesis Lectures on Data Management. Morgan & Claypool Publishers. DOI: 10.2200/S00873ED1V01Y201808DTM051. URL: <https://doi.org/10.2200/S00873ED1V01Y201808DTM051>.
-  Wood, Peter T. (2012). "Query languages for graph databases". In: *SIGMOD Rec.* 41.1, pp. 50–60. DOI: 10.1145/2206869.2206879. URL: <https://doi.org/10.1145/2206869.2206879>.