Data Processing and Analytics (DISS-DPA)

Principles of Data Quality – Data Quality Rules for Graphs

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Motivation

Previously

- Data quality is an important problem in data management
- Dirty data is everywhere and costly
- A principled approach to detect and repair errors
  - Using quality improving dependencies (QIDs)
  - Capturing conditional functional dependencies (CFDs), matching dependencies (MDs), etc.

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In this Episode

Can we transfer these techniques from the relational setting to graphs?

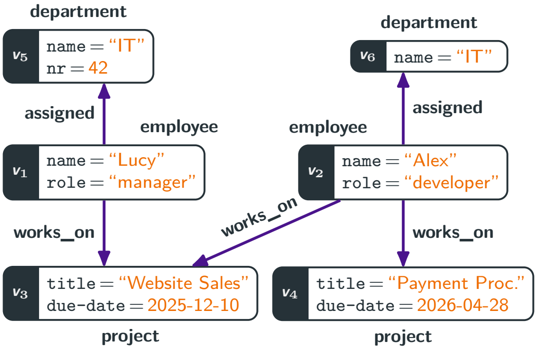
- Instead of a relational database consisting of tables, a database is represented by a graph

Data Model: Graphs with Labels and Properties

Data Model

In this chapter, we represent a database by directed graphs $G = (V, E, L, P)$ where

- V is a set of nodes
- E is a set of edges



Data Model: Graphs with Labels and Properties

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- V is a set of nodes
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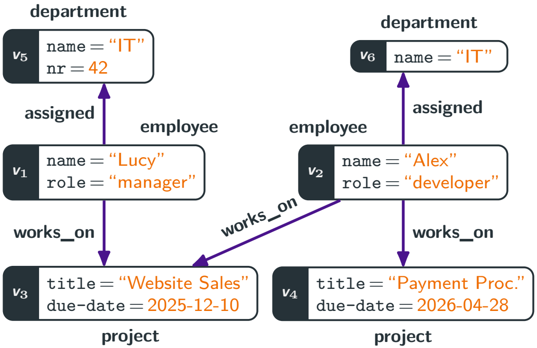
- L is a function that assigns
  - a label $L(v)$ to every node $v \in V$
  - a label $L(e)$ to every edge $e \in E$



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  - a label $L(v)$ to every node $v \in V$
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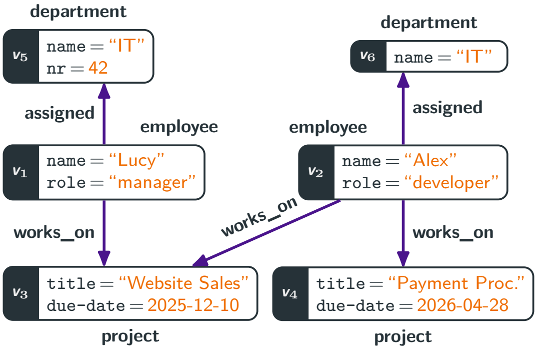
- P is a function that assigns
  - a set of key-value pairs, called properties, to every node $v \in V$



Recall: Ingredients for the Repair Problem

1. Quality dependencies
   - We considered quality improving dependencies
2. A dirty relational database
3. A repair model
   - What kind of operations are allowed to modify the database?
   - Examples: tuple deletions, tuple insertions, value modifications
4. A cost model
   - the repair should differ minimally
   - Examples: number of deletions, edit distance

Goal

A clean database that satisfies all the dependencies

Recall: Ingredients for the Repair Problem

1. Quality dependencies
   - We considered quality improving dependencies
   - Are they applicable to graphs?
2. A dirty graph database
3. A repair model
   - What kind of operations are allowed to modify the database?
   - Examples: ~~tuple deletions, tuple insertions~~, value modifications
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Goal

A clean graph that satisfies all the dependencies

Recall: Data Improving Dependencies

Recall: Data Improving Dependencies (QIDs)

Formalism for data quality rules that covers

- Functional Dependencies (FDs)
- Conditional Functional Dependencies (CFDs)
- Matching Dependencies (MDs)

Example (A CFD written as a QID)

"In the UK, the zip code uniquely determines the street"

$$\forall t_1 \forall t_2 \Big(\big(\text{Address}(t_1) \wedge \text{Address}(t_2) \wedge t_1[\text{zip}] = t_2[\text{zip}] \wedge t_1[\text{CC}] = t_2[\text{CC}] \wedge t_1[\text{CC}] = 44\big) \rightarrow t_1[\text{street}] = t_2[\text{street}] \Big)$$

Recall: Data Improving Dependencies

Quality Improving Dependency (QID)

A quality improving dependency (QID) is a first-order sentence of the following form

$$\forall t_1 \forall t_2 \Big(\big(R(t_1) \wedge S(t_2) \wedge \bigwedge_{i \in [1,n]} t_1[A_i] \ \mathrm{op}_i \ t_2[B_i] \big) \rightarrow \bigwedge_{j \in [1,m]} t_1[C_j] \ \mathrm{op}'_j \ t_2[D_j] \Big)$$

where the operators op_i and op'_j form the signature of the dependency

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Operators

- Equality: $t_1[A] = t_2[B]$ iff attribute A of t_1 and B of t_2 have the same value
- Equality with constant: $t_1[A] =_c t_2[B]$ iff attribute A of t_1 and B of t_2 have value c
- Similarity: $t_1[A] \sim t_2[B]$ iff the values of attribute A of t_1 and B of t_2 are similar relative to some similarity relation $\sim$

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Do QIDs Apply to Graphs?

- Attributes correspond to properties of nodes

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Do QIDs Apply to Graphs?

- Attributes correspond to properties of nodes
- But what about tuples? Do they correspond to nodes or edges, or something else?
- What about the labels of nodes and edges?
- Goal: compare nodes in specific subgraphs instead of tuples

Identifying Subgraphs with Graph Patterns

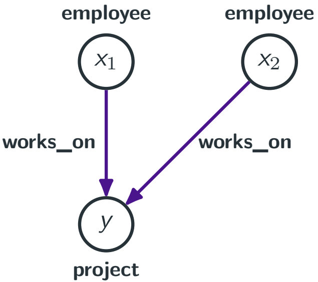
- We use graph patterns to identify subgraphs in our quality dependencies for graphs

Graph Pattern

A graph pattern is a tuple $Q = (X_Q, E_Q, L_Q)$ where

- (X_Q, E_Q) is a directed graph
- the nodes in X_Q are called variables
- L_Q is a function that assigns labels to nodes/variables and edges

Example (Graph Pattern)



All pairs of employees x_1, x_2 working on a common project y

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Note

Graph patterns do not refer to properties

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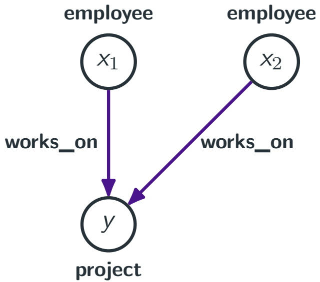

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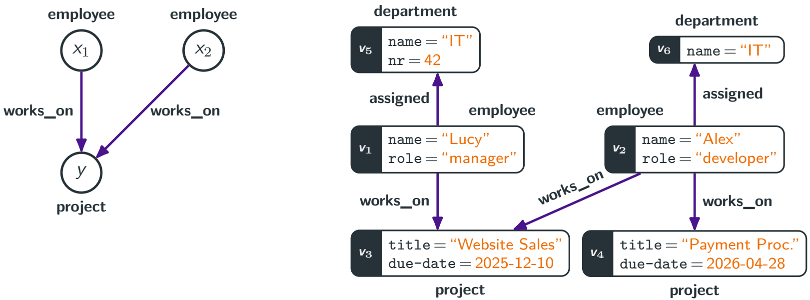
Matching Semantics



Match

A match of a pattern $Q = (X_Q, E_Q, L_Q)$ in a graph $G = (V, E, L, P)$ is a function $h \colon X_Q \to V$ such that

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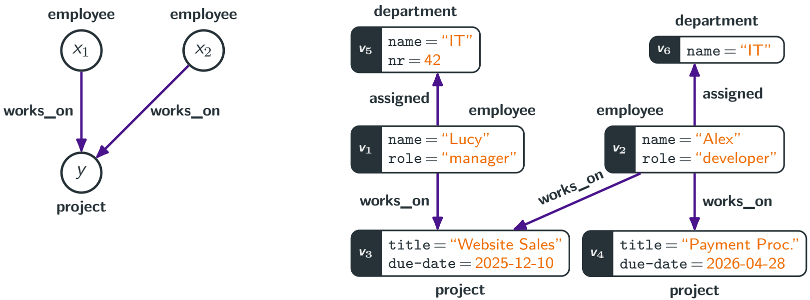

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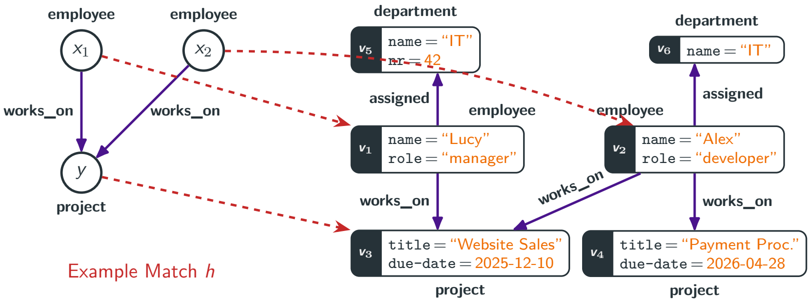
- $L_Q(x) = L(h(x))$ for all $x \in X_Q$
- For all edges $(x, y) \in E_Q$:
  - $(h(x), h(y)) \in E$ is an edge in G
  - $L_Q(x, y) = L(h(x), h(y))$



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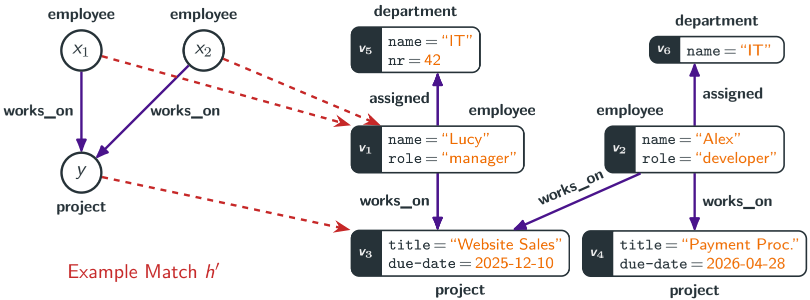
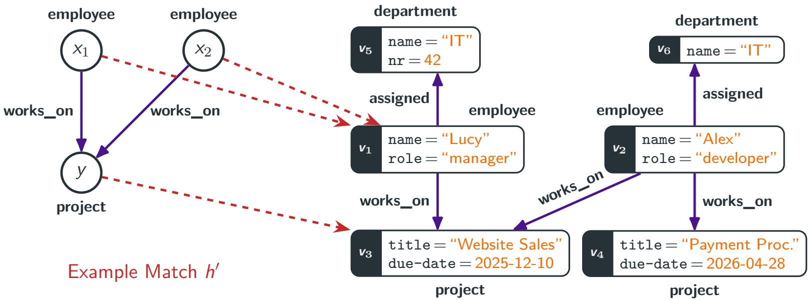


Notes

- Multiple variables can be mapped to the same node

Matching Semantics



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- We thus consider homomorphic matches

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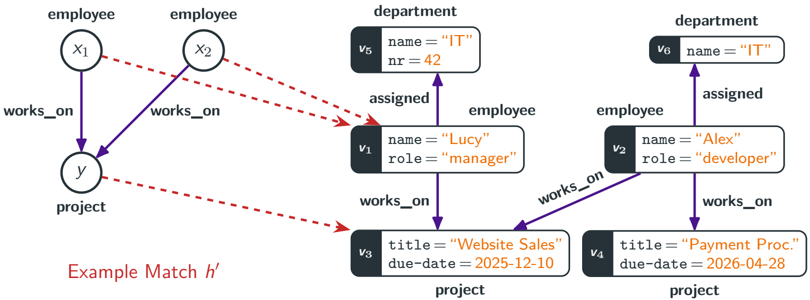

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- There are many alternative matching semantics: Big Graph Processing Systems course

Graph Entity Dependencies[1]

Graph Entity Dependencies (GEDs)

A graph entity dependency (GED) has the following form

$$Q\Big(\bigwedge_{i\in[1,n]} \varphi_i(x_i, y_i) \rightarrow \bigwedge_{j\in[1,m]} \psi_j(z_j, u_j)\Big)$$

- Q is graph pattern
- x_i, y_i, z_j, u_j are variables of Q
- φ and ψ are literals

[1] Fan and Lu, "Dependencies for Graphs", *Proceedings of the 36th ACM Symposium on Principles of Database Systems, PODS 2017*, 2017

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Literals

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- Node Equality: $x = y$ iff x and y represent the same node

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Semantics

A graph G satisfies a GED with graph pattern $Q = (X_Q, E_Q, L_Q)$ if

- for every match of h of Q in G:
  - if $(G, h(x_i), h(y_i)) \models \varphi_i(x_i, y_i)$ holds for all $i \in [1, n]$
  - then $(G, h(w_j), h(u_j)) \models \psi_j(x_j, y_j)$ holds for all $i \in [1, m]$

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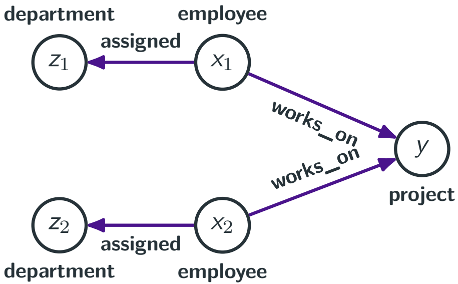
Graph Entity Dependencies – Example

Example

Graph Pattern Q_1



Graph Entity Dependency

$$Q_1\Big(\text{true} \rightarrow z_1 = z_2\Big)$$

If two employees work on the same project, they are assigned to the same department

Example

Graph Pattern Q_2

Graph Entity Dependency

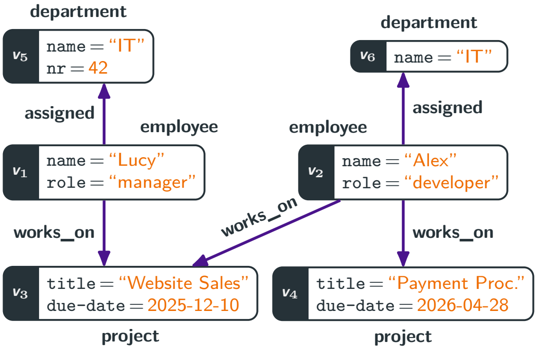
$$Q_2\Big(z_1[\text{name}] = z_1[\text{name}] \rightarrow z_1[\text{nr}] = z_1[\text{nr}]\Big)$$

If two departments have the same name,
they also have the same number

Graph Entity Dependencies – Example

Graph Entity Dependencies

If two employees work on the same project, they are assigned to the same department

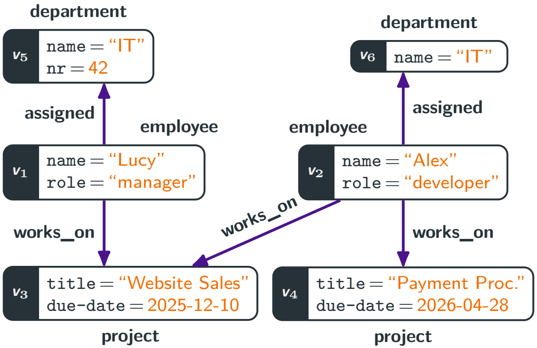


Graph Entity Dependencies – Example

Graph Entity Dependencies

If two employees work on the same project, they are assigned to the same department

- Not satisfied by G ✗



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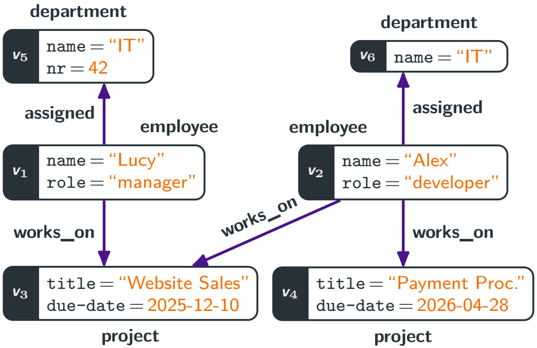

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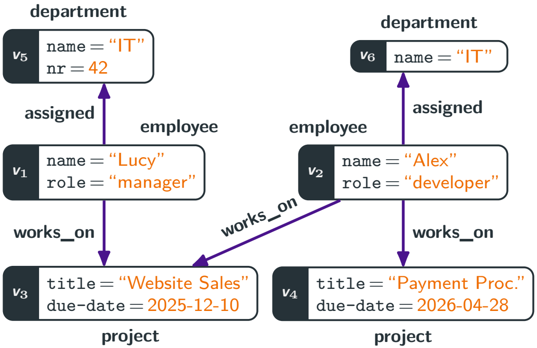

GEDs vs. QIDs

GEDs vs. CFDs

CFDs can be translated into GEDs

- Every tuple in the database is interpreted as a node
  - labelled with the relation it belongs to
- The graph pattern of the GED consists of two disconnect nodes labelled with R and S

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GEDs vs. MDs

- GEDs do not support similarity operators
- and can therefore not mimic
- But they can be extended accordingly

Ingredients for the Repair Problem

1. Quality dependencies ✔
   - We consider Graph Entity Dependencies
2. A dirty graph database ✔
3. A repair model
   - What kind of operations are allowed to modify the database?
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Goal

A clean graph that satisfies all the dependencies

Repair Model

- We discuss a variation of the V-Repair model, adapted for graphs
- It allows for changing values of properties

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- It allows for changing values of properties
- It allows for merging nodes
  - If two nodes v_1, and v_2 are merged into a new node w,
  - then w inherits all outgoing edges from v_1 and v_2,
  - and all incoming edges of v_1 and v_2 are redirected to w

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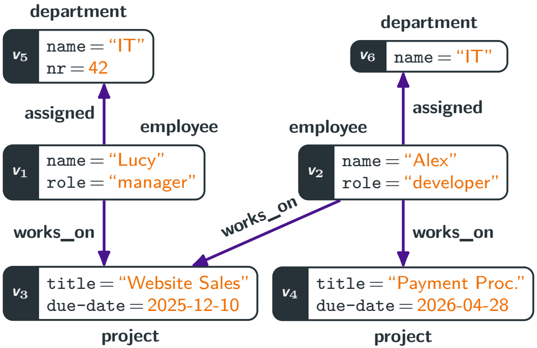
  - then w inherits all outgoing edges from v_1 and v_2,
  - and all incoming edges of v_1 and v_2 are redirected to w
- Labels and edges cannot be changed directly

Repair Model – Example

Graph Entity Dependency

> *If two employees work on the same project, they are assigned to the same department*

- Not satisfied by G ✗



Repair Model – Example

Graph Entity Dependency

If two employees work on the same project, they are assigned to the same department

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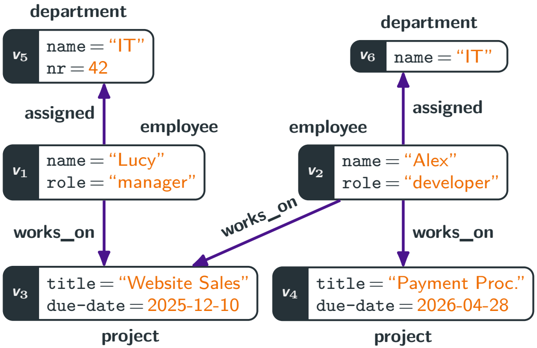
- To repair G, we merge v_5 and v_6



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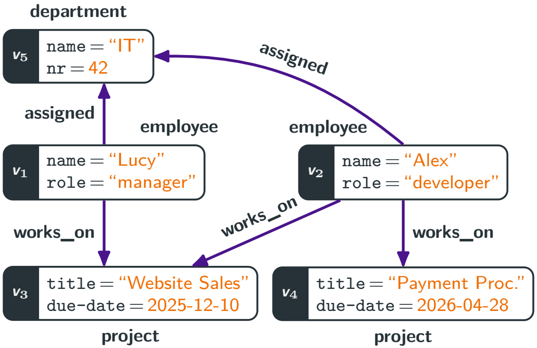

Ingredients for the Repair Problem

1. Quality dependencies ✔
   - We consider Graph Entity Dependencies
2. A dirty graph database ✔
3. A repair model ✔
   - Modification of property values, merging of nodes
4. A cost model ✔
   - the repair should differ minimally
   - Examples: number of merges, edit distance

Goal

A clean graph that satisfies all the dependencies

Chasing Graphs

Recall: Chase Procedures

Idea

- We adapt the (extended) chase procedure for graphs and our repair model

Recall: Chase Procedures

The chase takes as input

- a set Σ of data quality rules; and
- an input database D,

and, if the chase terminates successfully, then it outputs a database D' such that $D' \models \Sigma$

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The chase takes as input

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Recall: Implementation

- The chase procedure can fire a dependency σ if σ is not satisfied
- σ is fired for a specific violation, which is then repaired (unless there is a conflict)

The Chase for GEDs

Let

$$\sigma = Q\Big(\bigwedge_{i \in [1,n]} \varphi_i(x_i, y_i) \rightarrow \psi(z, u) \Big)$$

be a graph entity dependency (GED) with graph pattern $Q = (X_Q, E_Q, L_Q)$

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Firing of a GED

The GED σ can be fired on a graph G if there is a match h of Q in G such that

- $(G, h(x_i), h(y_i)) \models \varphi_i$ holds for all $i \in [1, n]$
- but $(G, h(z), h(u)) \models \psi(z, u)$ does not hold

The Chase for GEDs

The Chase Procedure for GEDs

1. Initialize $G' = G$
2. As long as there is
   - a GED $\sigma \in \Sigma$ with consequence $\psi(z, u)$,
   - and a match h

   for which σ can be fired do

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The Chase for GEDs

The Chase Procedure for GEDs

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   - a GED $\sigma \in \Sigma$ with consequence $\psi(z, u)$,
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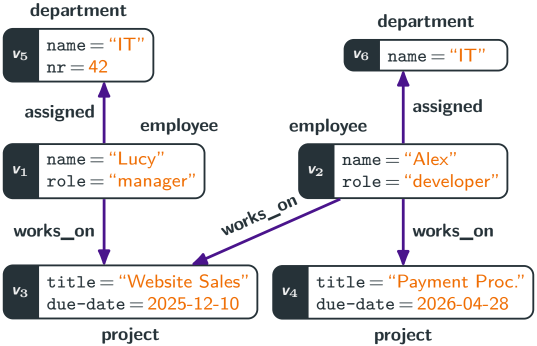
   2.5 If none of the other cases applies, abort

The Chase for GEDs – Example

Graph Entity Dependencies

- GED σ_1:
  If two departments have the same name, they also have the same number

- GED σ_2:
  If two employees work on the same project, they are assigned to the same department



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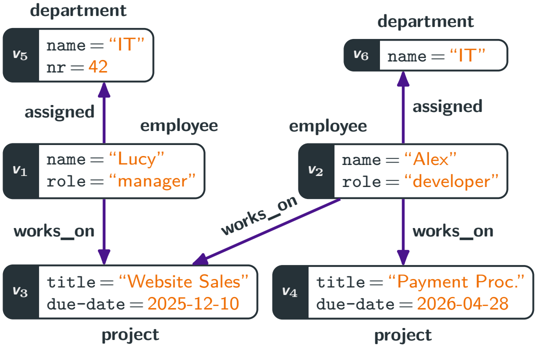

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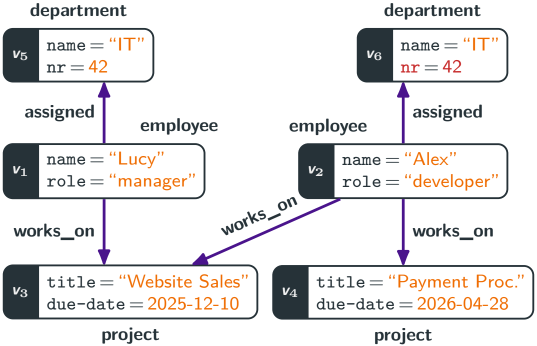

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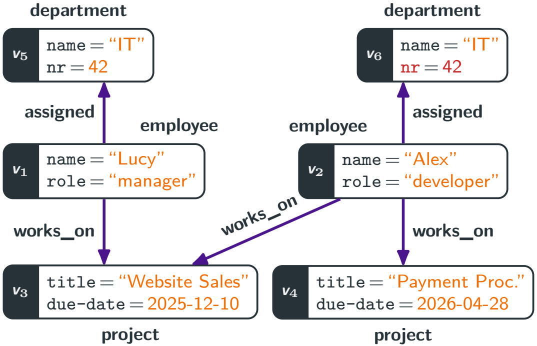

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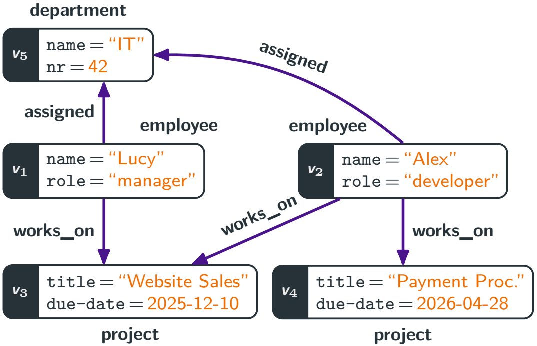

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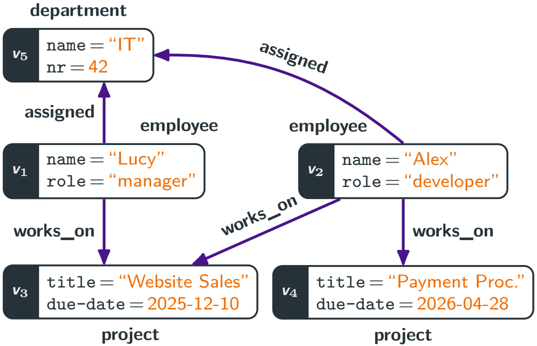

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   - Merge v_5 and v_6
3. No GED can be fired
   - The chase terminates successfully

Properties of the Chase for Graphs and GEDs

The chase procedure for graphs and GEDs has two important properties

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Chasing with GEDs has the Church-Rosser Property

For all graphs G, and a sets Σ of GEDs, the chase either

- always returns the same repair for G and Σ; or
- always aborts due to a conflict

regardless in which order and for which errors GEDs are fired

Reasoning about GEDs

Reasoning about GEDs: The Satisfiability Problem

The Satisfiability Problem

Input: A finite set Σ of GEDs

Question: Is there a non-trivial graph G that satisfies all dependencies in Σ?

Goal

Automatically Verifying that a set of GEDs is consistent

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- But in PTIME, if the relational schema does not enforce finite domains for attributes!
- Since our graphs have no schema, the difficulty is not inherited from CFDs

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Input: A finite set Σ of GEDs, a GED σ

Question: Does Σ imply σ?

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Theorem (Fan and Lu, "Dependencies for Graphs", Theorem 5.12)

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Reasoning about GEDs: The Error Detection Problem

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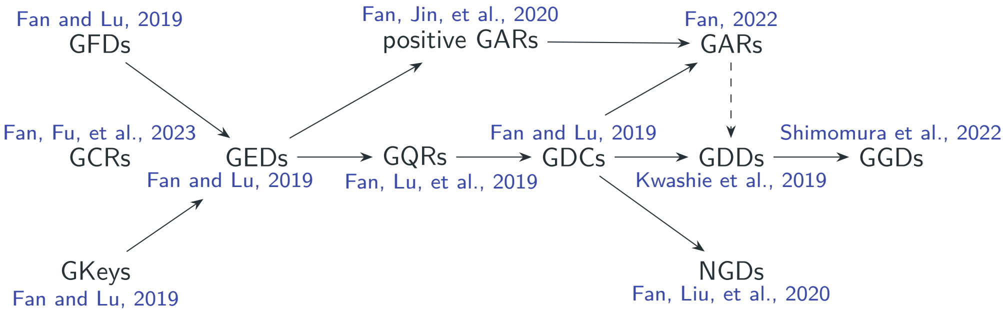
Theorem (Fan and Lu, "Dependencies for Graphs", Theorem 5.16)

The error detection problem for GEDs is NP-complete, even if G is a tree

- For CFDs in the relational setting, the error detection problem is in PTIME

The Frontier of Graph Repairs

A Family of Graph Dependencies



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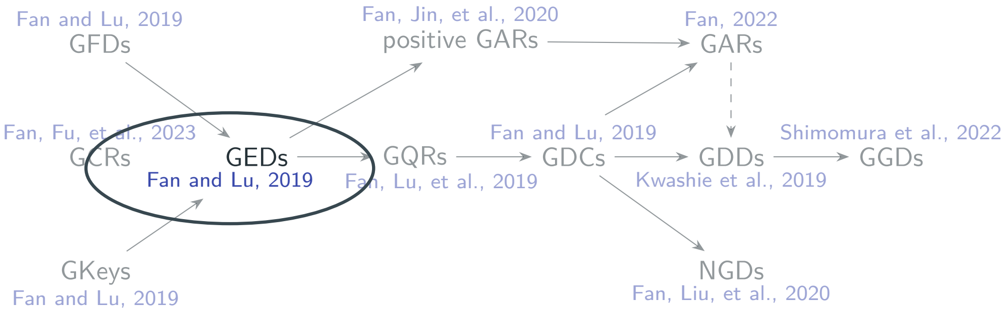
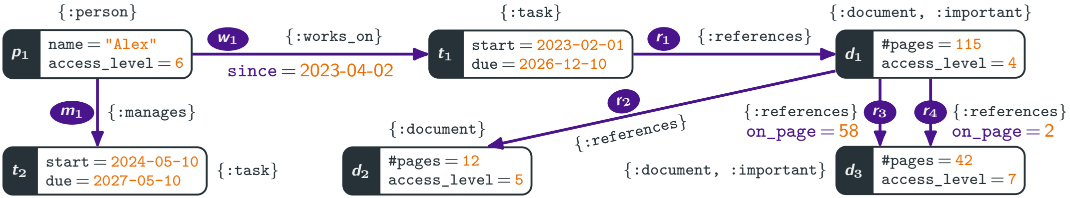


Outlook: Property Graphs



Property Graphs

- Nodes and edges can have multiple labels
- Multi-graphs: there can be more than edge between two nodes
- Nodes and edges can have properties

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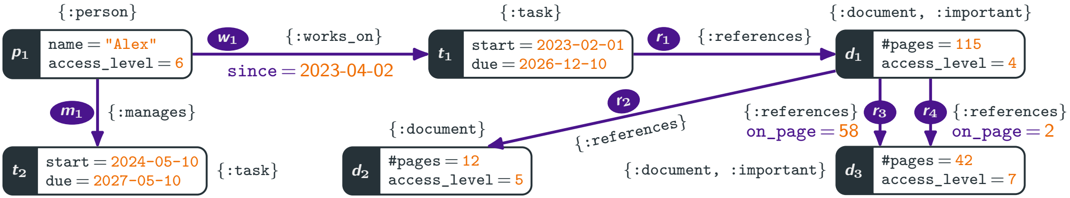

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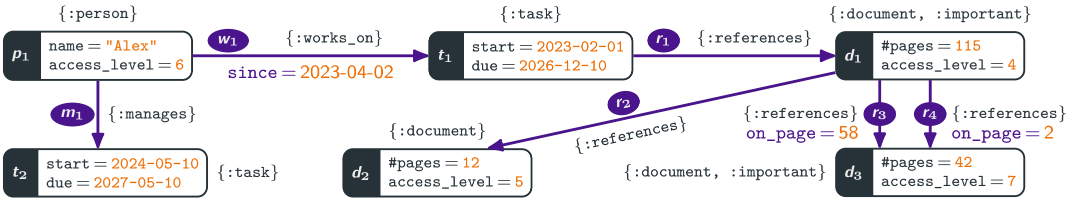

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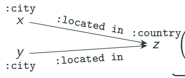
- More in the “Big Graph Processing Systems” course in January/February 2026

Outlook: GQL and PG-Constraints

GDCs

and related notions



$$\left(x \neq y \quad \Rightarrow \quad x.\texttt{name} \neq y.\texttt{name}\right)$$

Pattern with variables $x, y, z, \ldots$

Dependency $X \Rightarrow Y$

PG-Constraints

```
FOR x, y, z
MATCH (x:city)-[:located in]->(z:country),
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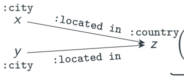

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Pattern with variables $x, y, z, \ldots$ — Dependency $X \Rightarrow Y$

- ✔ The satisfiability, validation, and implication problems have been studied[2]
- ✔ Data cleaning has been studied[3] for GDCs without $\leq$ and $\geq$
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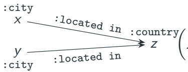

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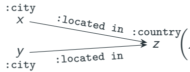

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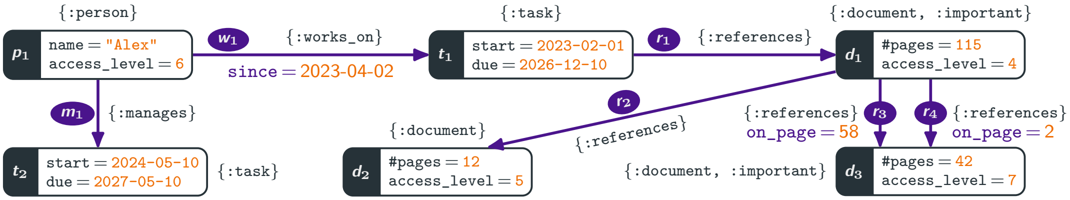

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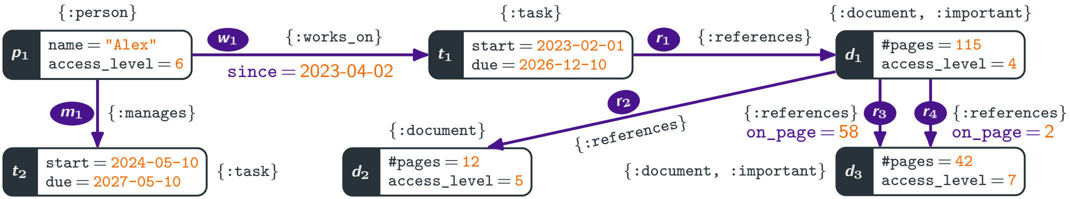

Constraint
If a person works on a task, which has started and which references directly or indirectly, an important document, then the person's access level is at least as high as the (required) access level of the referenced document.

- This constraint cannot be described by a GED, since it talks about arbitrary length paths

Summary

Data Quality for Graphs

- Graph entity dependencies (GEDs) are a formalism for data quality rules for graphs
  - They cover CFDs from the relational setting (and more)
- Graph patterns are used to identify subgraphs
- The chase can be adapted for GEDs
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Take away message

Data quality for graphs: a rich source of problems and challenges

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